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MATHEMATICS 7

Number Theory

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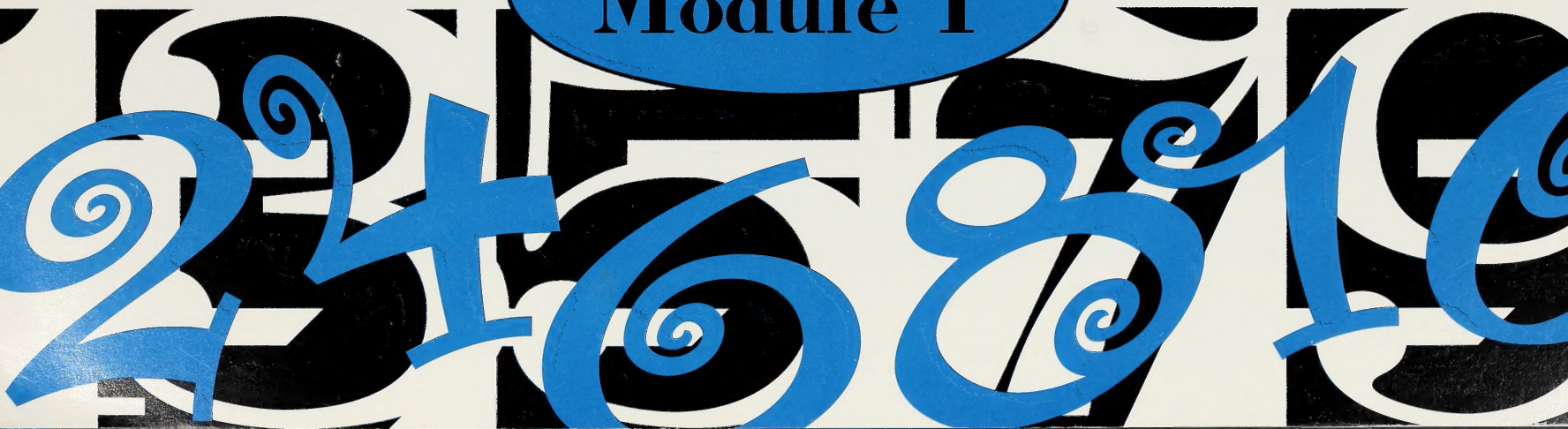
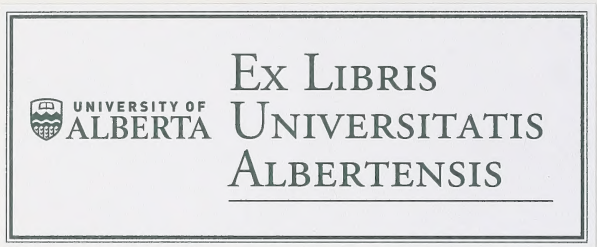
Module 1

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Module 1

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Mathematics 7
Student Module Booklet
Module 1
Number Theory
Alberta Distance Learning Centre
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Teachers (Mathematics 7)	✓
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Welcome

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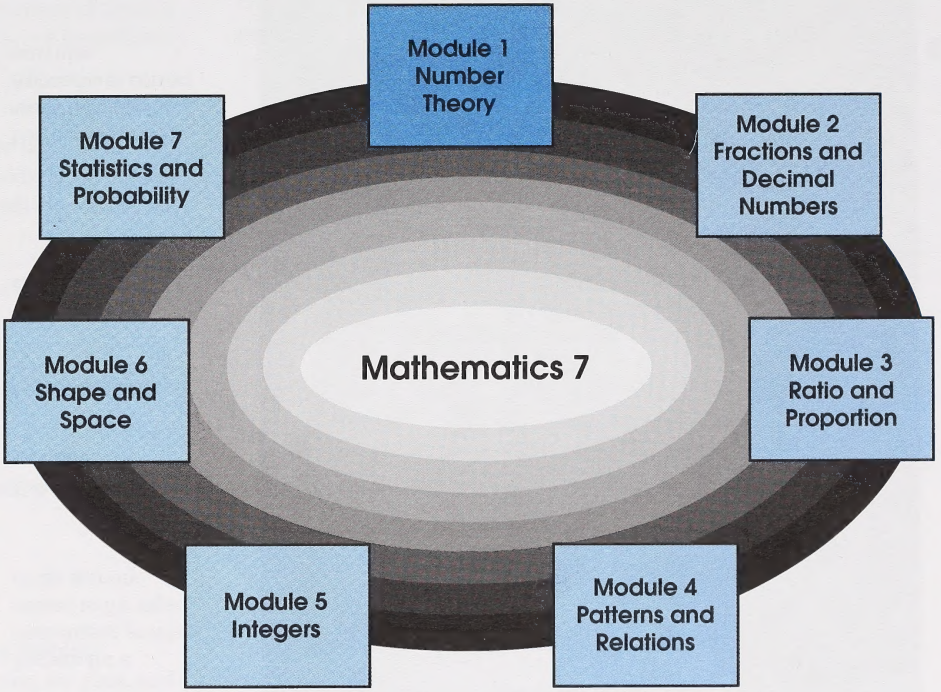
Welcome to Module 1. We hope you'll enjoy your study of Number Theory.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

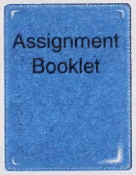
- Use computer software.

- Use a scientific calculator.

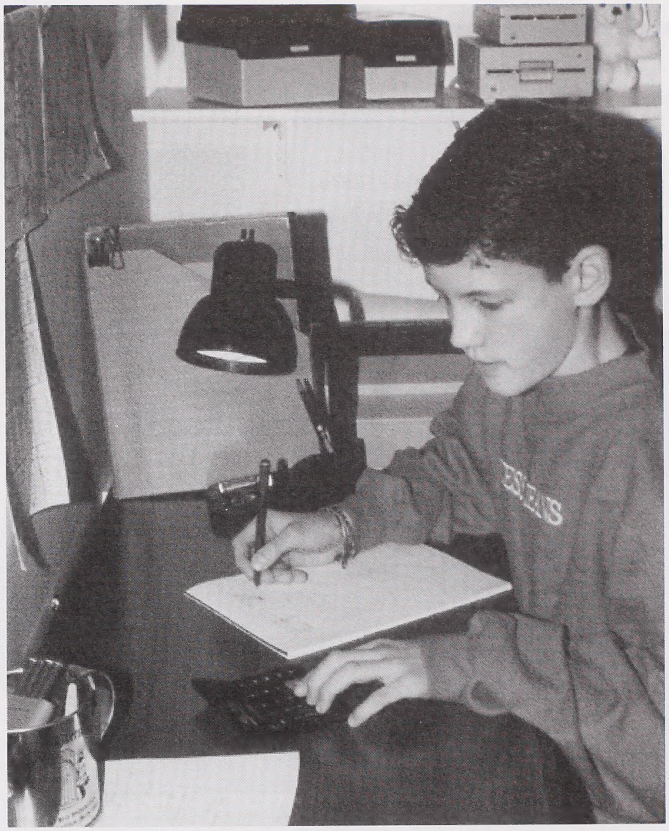
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The icon is a cue that the problem will be related to the topic of the activity.

The icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in this module explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to this Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

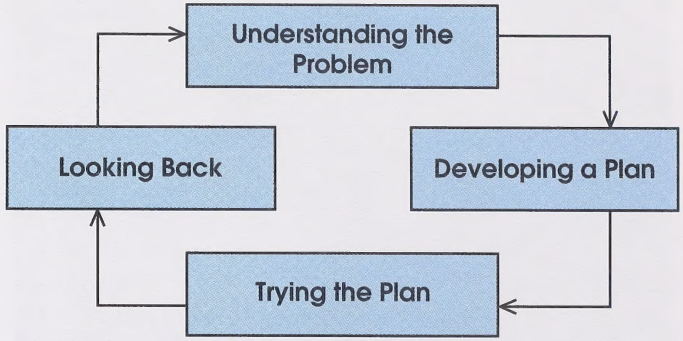
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

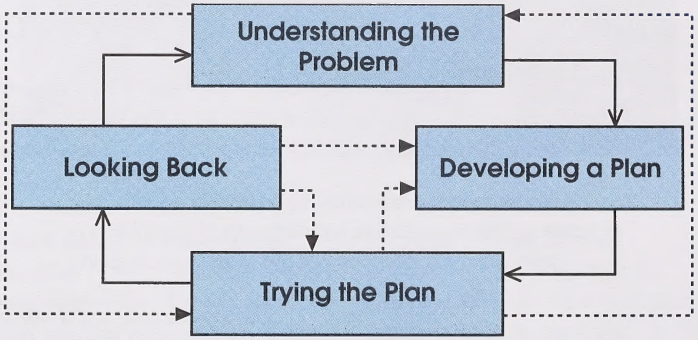
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



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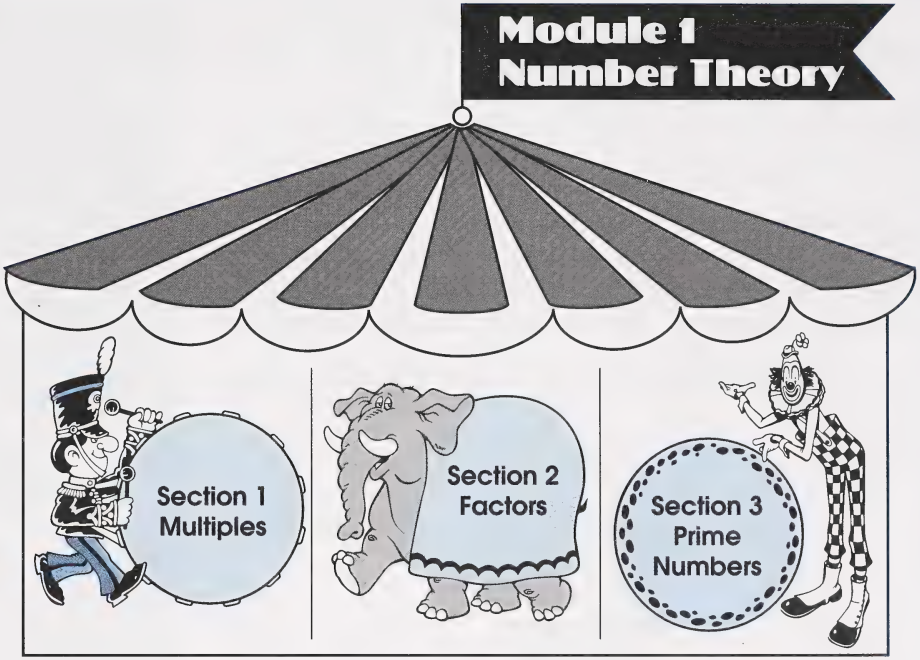
Module Overview

Imagine a world without numbers. What would it be like? How could you tell anyone how old you are or how tall you are? How could you give your address? Numbers are needed for the purposes of counting, locating, measuring, and communicating. Numbers can also be fascinating and fun.

As a child you were curious about numbers and their meaning. You saw numerals in many places: on houses, football jerseys, clocks, telephones, and money. Over the years your number sense and skills with numbers improved.

In this module you will explore number theory, a branch of mathematics that has intrigued mathematicians for centuries. You will discover more about the properties of numbers as you work with multiples, factors, powers, and prime numbers.

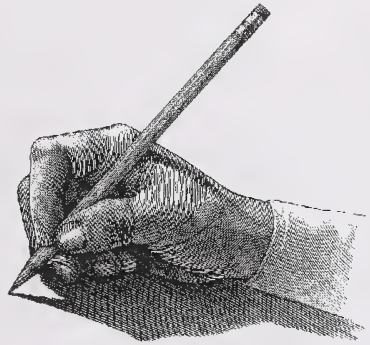


Evaluation

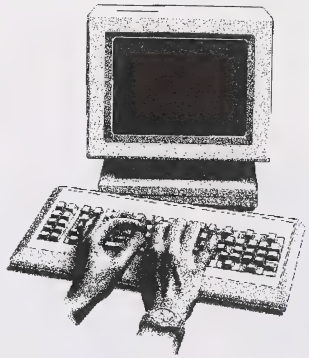
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete four assignments. The mark distribution is as follows:

Section 1 Assignment	**25 marks**
Section 2 Assignment	**25 marks**
Section 3 Assignment	**25 marks**
Final Module Assignment	**25 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Multiples

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Multiple births are common among dogs and cats, but they are rarer in humans. You probably know people who are twins, but you likely do not know anyone who is one of a triplet, quadruplet, or quintuplet.

In mathematics, as you will discover, the word *multiple* has a specialized meaning.

In this section, you will list the multiples of a number, find the common multiples and the least common multiple of two or more numbers, and use Venn diagrams to sort numbers.

Activity 1: Multiples

In elementary grades you learned to skip count. For example, you learned to count by 3s.

3, 6, 9, 12,…

What you were in fact doing is listing the **multiples** of 3.

A multiple of a number is found by multiplying the number and another whole number, or by adding the number repeatedly.

The multiples of 3 are 3, 6, 9, 12,…

+3 +3 +3

3×1 3×2 3×3 3×4

1. **a.** List the first five multiples of 4. That is, count by 4s.
b. List the first five multiples of 5. That is, count by 5s.

Check your answers by turning to the Appendix.

Using a Calculator to Find Multiples

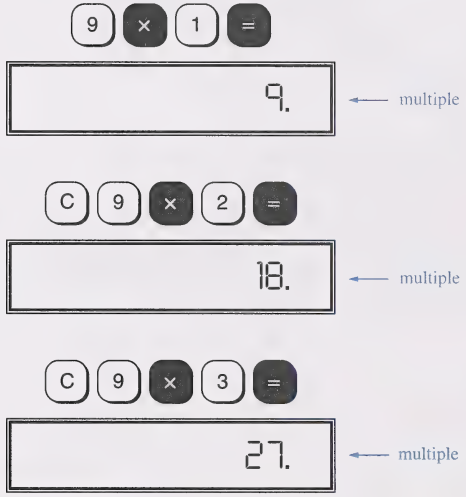
You can use a calculator to find the multiples of a number. There are many different ways to find multiples, as the following example illustrates.

Example

Use your calculator to find the first three multiples of 9.

Solution

Method 1: Using Multiplication



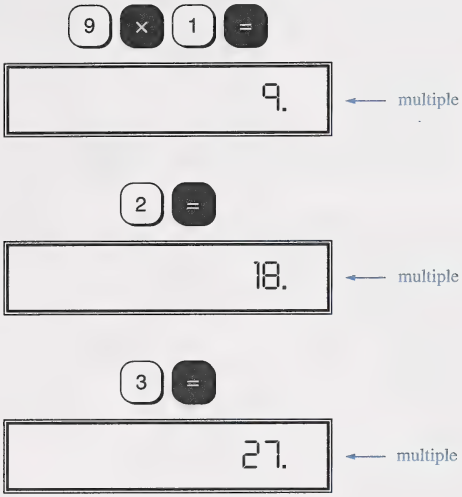
The first three multiples of 9 are 9, 18, and 27.

Note: Pressing C clears the display.

Method 2: Using the Multiplication Constant Feature

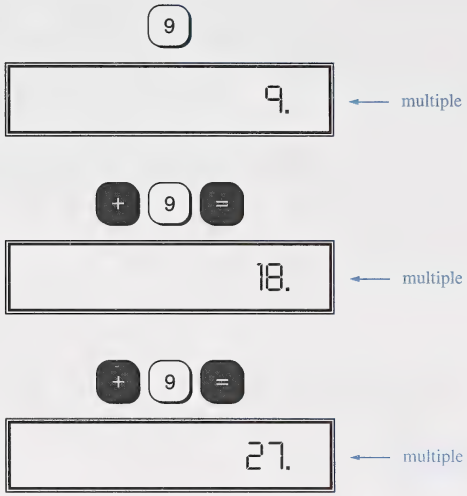
Some calculators have a built-in multiplication constant. This feature allows the user to multiply repeatedly and reduces the number of key presses. However, the key sequence to use the built-in multiplication constant is not the same on every calculator. Experiment with your calculator and read the owner's manual to see if your calculator has this feature and how it works.

For some calculators, the following key sequence will give you the first three multiples of 9.



The first three multiples of 9 are 9, 18, and 27.

Method 3: Using Addition

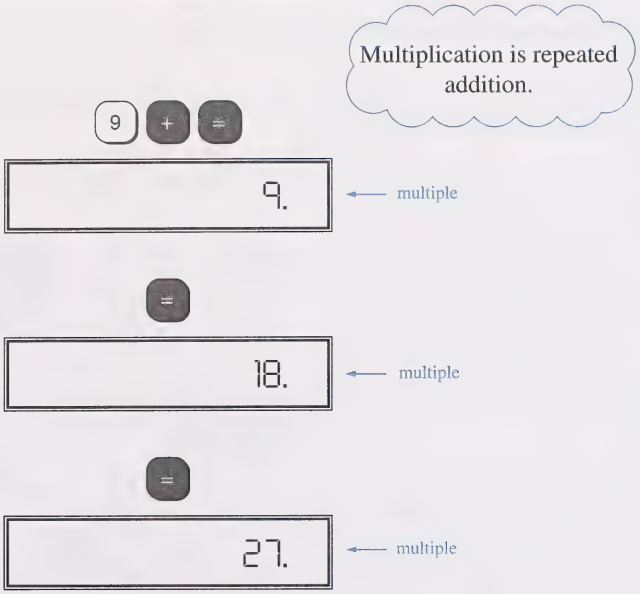


The first three multiples of 9 are 9, 18, and 27.

Method 4: Using the Addition Constant Feature

Some calculators have a built-in addition constant. You may use this feature to add repeatedly. Like the multiplication constant feature, the addition constant feature reduces the number of key presses.

The following key sequence will give you the first three multiples of 9.



The first three multiples of 9 are 9, 18, and 27.

Use a calculator to answer the following questions.

2. List the first five multiples of each of these numbers.

a. 8 **b.** 12 **c.** 23 **d.** 32

3. Is the first number a multiple of the second number in each of the following? Answer **yes** or **no**.

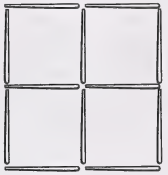
a. 94, 7 **b.** 86, 4 **c.** 98, 12
d. 56, 14 **e.** 35, 7 **f.** 49, 6

Check your answers by turning to the Appendix.

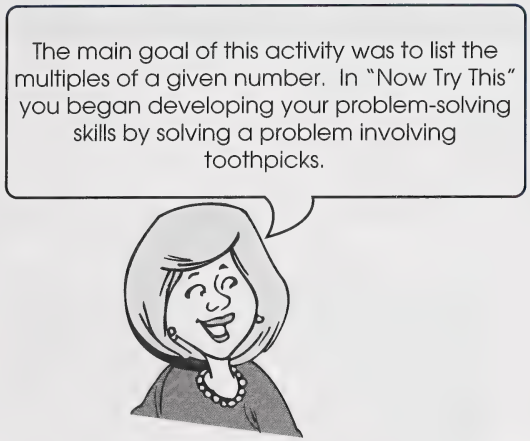
Now Try This

Use a problem-solving strategy to answer the following question.

4. In the sketch 12 toothpicks are arranged to make 4 squares of equal size. How can you make 3 squares of equal size by repositioning 3 toothpicks? **Hint:** Each toothpick must be used as a side of a square.



Check your answer by turning to the Appendix.


The main goal of this activity was to list the multiples of a given number. In "Now Try This" you began developing your problem-solving skills by solving a problem involving toothpicks.

Activity 2: Common Multiples

Sally and Kim are best friends. They have many common interests —pets, music, and volunteering at the hospital. What common interests do you share with your best friend?

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In this activity you will find the **common multiples** of two or more numbers and identify the **least common multiple (LCM)**.

When a number is a multiple of two or more numbers it is called a common multiple. The least common multiple (LCM) of two or more numbers is the least of the common multiples of the numbers.

Example

What are three common multiples of 20 and 30? What is the least common multiple (LCM)?

Solution

Step 1: List the multiples of 20.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200,…

Step 2: List the multiples of 30.

30, 60, 90, 120, 150, 180,…

Step 3: Circle the common multiples of 20 and 30.

20, 40, (60), 80, 100, (120), 140, 160, (180), 200,…

30, (60), 90, (120), 150, (180),…

Three common multiples of 20 and 30 are 60, 120, and 180.

Step 4: Put a square around the least common multiple of 20 and 30.

20, 40, (60), 80, 100, (120), 140, 160, (180), 200,…

30, (60), 90, (120), 150, (180),…

The least common multiple of 20 and 30 is 60.

Note: There is an infinite number of common multiples of two numbers. There is only **one** least common multiple of the two numbers.

1. **a.** List the first ten multiples of 6.
b. List the first ten multiples of 9.

2. Use the lists you generated in question 1 to answer 2.a. and 2.b.

a. Circle the common multiples of 6 and 9.
b. Put a square around the least common multiple of 6 and 9.

3. Give the first three common multiples and the least common multiple for each of the following pairs of numbers.

a. 4 and 6
b. 8 and 12
c. 18 and 24

4. Find the LCM for each of these sets of numbers.

a. 4, 6, and 10
b. 16, 18, and 24

5. In training for a long distance race, Camilla and Karen complete several laps of a track. Camilla's average time to complete one lap of the track is 9 min. Karen's average time to complete one lap is 12 min.

Min is the symbol for minutes.

a. If the girls begin at the same time, what is the least amount of time in which they could cross the finish line together?

b. If the girls begin at the same time and they cross the finish line together, who will have completed more laps? How many more?

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6. Majeed waters the plants in his father's restaurant. One plant is watered every 8 days. Another plant is watered every 12 days.

a. If Majeed begins watering both plants on January 1, what is the next date both plants will be watered on the same day?

b. How many times in the year will both plants be watered on the same day?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

7. A can of pop costs $0.80 in a vending machine. The machine accepts quarters, nickels, and dimes, but it does not give change. Make a list of all the possible combinations of coins you can use to buy one can of pop.

Check your answer by turning to the Appendix.

In this activity you listed the common multiples of two or more numbers and identified the least common multiple (LCM). You also used one of the problem-solving strategies discussed in the Appendix of this module to solve a problem about a vending machine.

Activity 3: Venn Diagrams

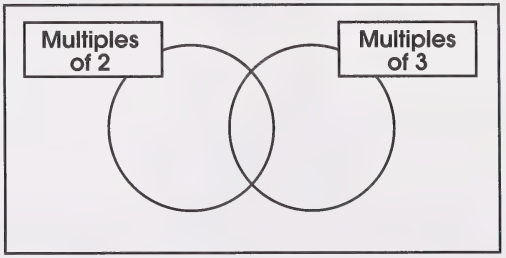
People are always inventing ways to sort things. An English mathematician named John Venn (1834–1923) devised a type of diagram called a **Venn diagram** that is helpful when sorting numbers.

A Venn diagram is a special kind of diagram in which circles are used to represent sets of numbers. The positioning of the circles in relation to one another represents the relationship among the sets.

Venn diagrams with two circles are useful in sorting numbers.

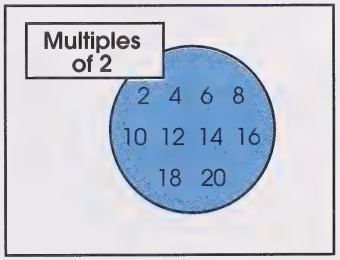
Example 1

Sort the numbers from 1 to 20 using the following Venn diagram.

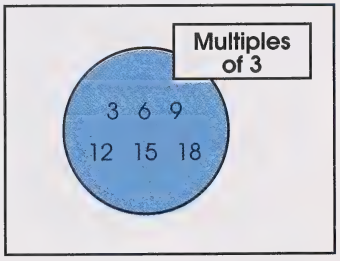


Solution

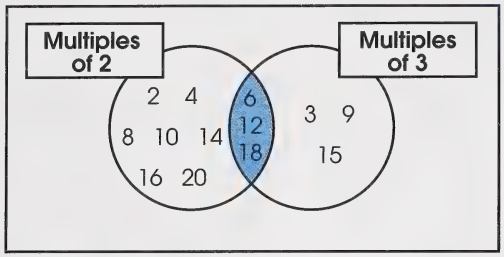
Step 1: Draw a circle and label it "Multiples of 2." Put the numbers from 1 to 20 that are multiples of 2 in the circle.



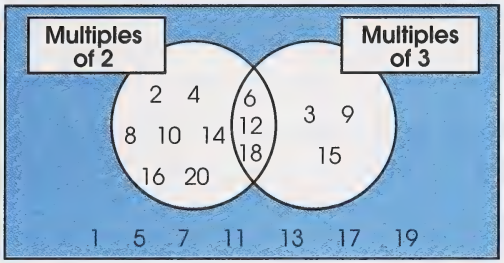
Step 2: Draw another circle and label it "Multiples of 3." Put the numbers from 1 to 20 that are multiples of 3 in the circle.

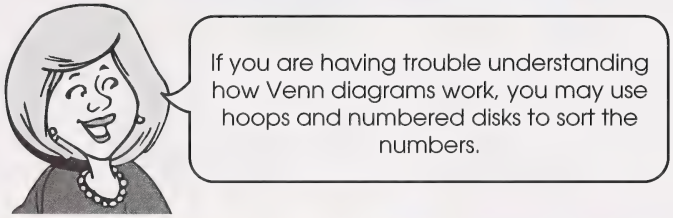


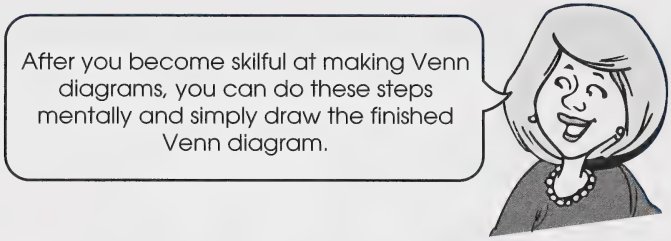
Step 3: Draw a diagram in which the circles overlap. Put the common multiples of 2 and 3 in the intersection of the circles (the area where the circles overlap).



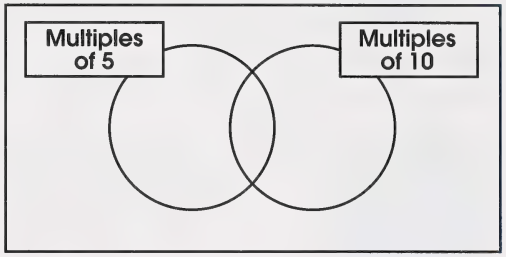
Step 4: Put the numbers from 1 to 20 that are not multiples of 2 or 3 outside the circles.



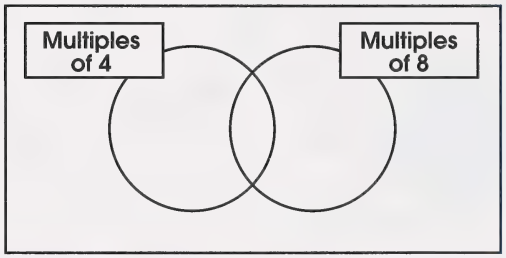



After you become skilful at making Venn diagrams, you can do these steps mentally and simply draw the finished Venn diagram.

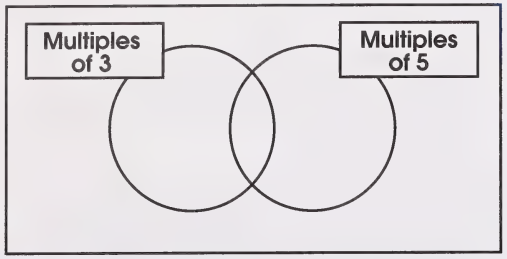
1. Complete a Venn diagram like the following for the numbers 1 to 20.



2. Complete a Venn diagram like the following for the numbers 1 to 20.



3. Complete a Venn diagram like the following for the numbers 1 to 20.

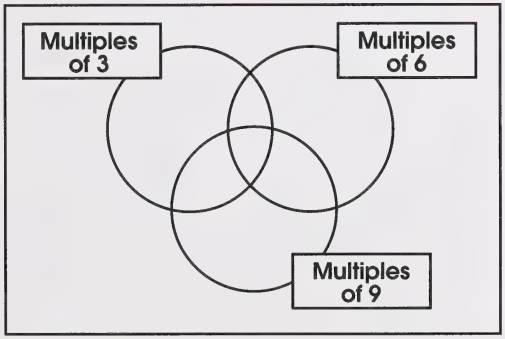


Check your answers by turning to the Appendix.

Venn diagrams with three circles are useful in sorting numbers.

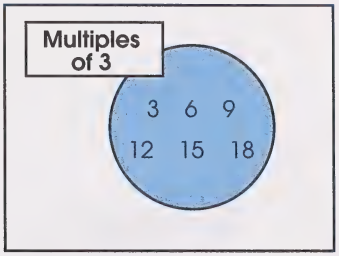
Example 2

Sort the numbers from 1 to 20 using a Venn diagram like the one shown.

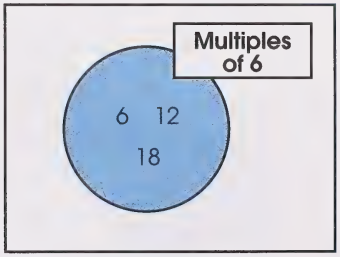


Solution

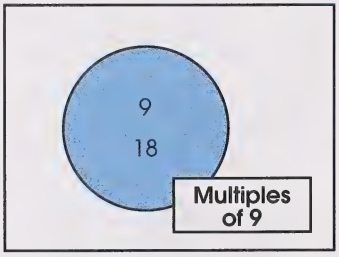
Step 1: Draw a circle and label it "Multiples of 3." Put the numbers from 1 to 20 that are multiples of 3 in the circle.



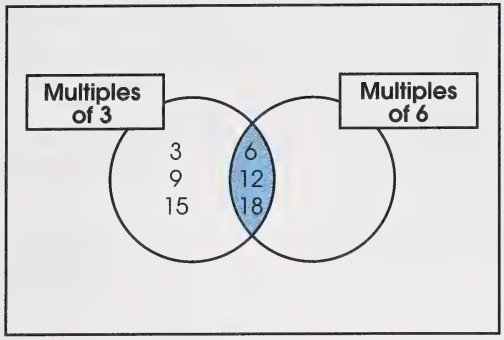
Step 2: Draw a second circle and label it "Multiples of 6." Put the numbers from 1 to 20 that are multiples of 6 in the circle.



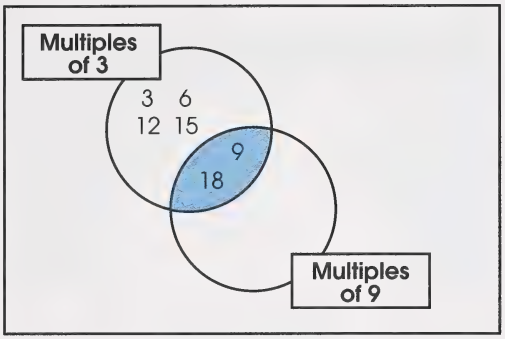
Step 3: Draw a third circle and label it "Multiples of 9." Put the numbers from 1 to 20 that are multiples of 9 in the circle.



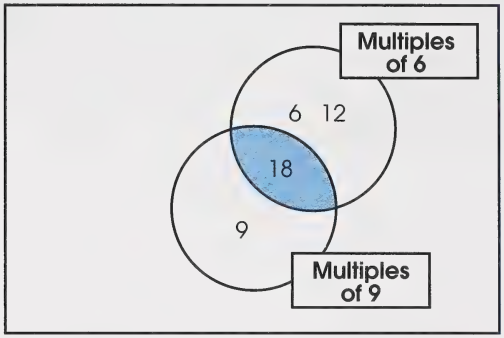
Step 4: Draw a diagram in which the circles for multiples of 3 and multiples of 6 overlap. Put the common multiples of 3 and 6 in the intersection.



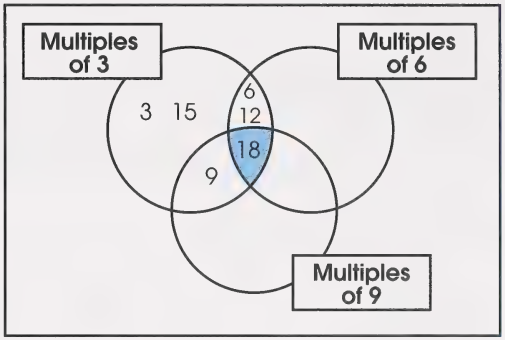
Step 5: Draw another diagram in which the circles for multiples of 3 and multiples of 9 overlap. Put the common multiples of 3 and 9 in the intersection.



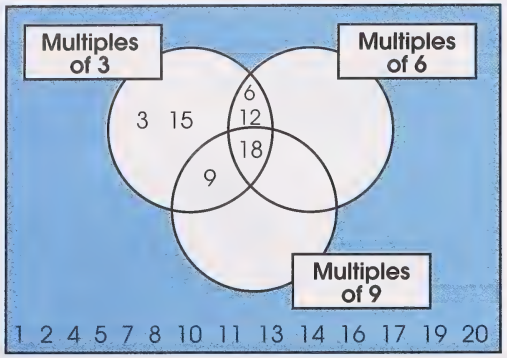
Step 6: Draw a third diagram in which the circles for multiples of 6 and multiples of 9 overlap. Put the common multiples of 6 and 9 in the intersection.

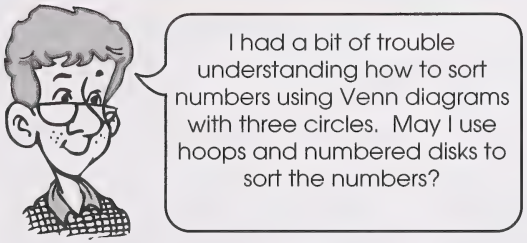


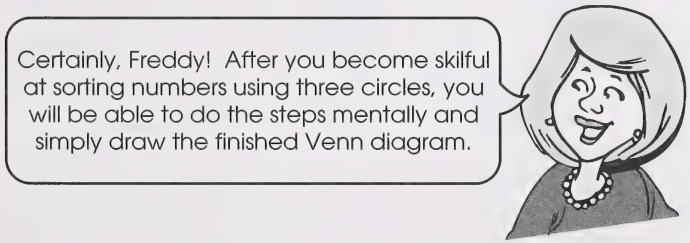
Step 7: Draw a final diagram in which the circles for multiples of 3, multiples of 6, and multiples of 9 overlap. Put the common multiples of 3, 6, and 9 in the intersection.



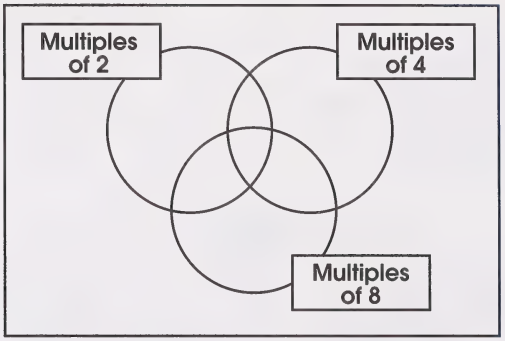
Step 8: Complete the final diagram; put the numbers from 1 to 20 that are not multiples of 3, 6, or 9 outside the circles.



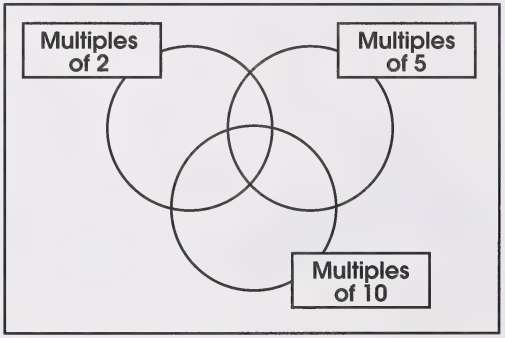

I had a bit of trouble understanding how to sort numbers using Venn diagrams with three circles. May I use hoops and numbered disks to sort the numbers?


Certainly, Freddy! After you become skilful at sorting numbers using three circles, you will be able to do the steps mentally and simply draw the finished Venn diagram.

4. Sort the numbers from 1 to 20 using the following Venn diagram.



5. Sort the numbers from 1 to 20 using the following Venn diagram.



Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

6. Using each of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 only once, fill in the squares to make this addition question true.

$$\square\square\square + \square\square\square = \square\square\square\square$$

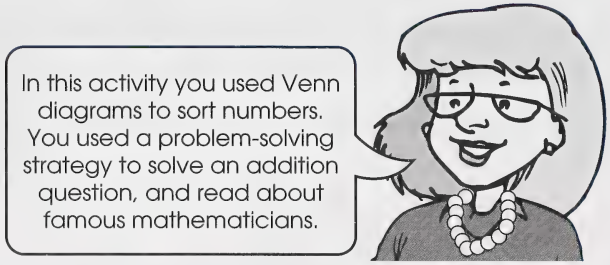
Check your answer by turning to the Appendix.

Did You Know?

The Internet has many sites devoted to the history of mathematics. Use the Internet to find out about these early women of mathematics: Hypatia (370–415), Maria Gaetana Agnesi (1718–1799), Sophie Germain (1776–1831), Mary Fairfax Somerville (1780–1872), Sofia Vasilyevna Kovalevskaya (1850–1891), Amalie Emmy Noether (1882–1935).

Hint: The MacTutor History of Mathematics archive on the World Wide Web has the biographies of hundreds of mathematicians. It is rated in the top 5% of all web sites. This is its uniform resource locator (URL):

http://www-groups.dcs.st-and.ac.uk/~history/index.html


In this activity you used Venn diagrams to sort numbers. You used a problem-solving strategy to solve an addition question, and read about famous mathematicians.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Using Models

Models are useful to help you visualize mathematical concepts.

View the segments entitled "Multiples," "Common Multiples," and "Least Common Multiples" of the program *Number Theory* from the series *Math Moves* and do the video activities.

Note: You will need blocks for the activity. The introduction to the video gives you ideas for gathering the blocks.

Check your answers by viewing the solutions given in the video.

Using Multiple Boards

You can use **multiple boards** to help you find the LCM.

A multiple board is a learning aid that has the first ten multiples of a number on it.

There are multiple boards in the Appendix. Photocopy the multiple boards, glue the multiple boards to heavy paper, and cut them out.

Example

Use multiple boards to find the LCM of 4 and 6.

Solution

Line up the multiple board for 4 and the multiple board for 6 so that the least common multiple on each board is across from the other.

6	12	18	24	30	36	42	48	54	60

4	8	12	16	20	24	28	32	36	40

LCM

The LCM of 4 and 6 is 12.

Use the multiple boards in the Appendix to answer the following questions.

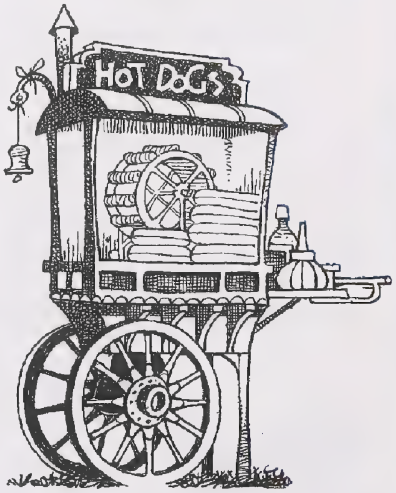
1. Find the LCM for each of the following pairs of numbers.

a. 2 and 3	**b.** 6 and 10	**c.** 6 and 8
d. 4 and 12	**e.** 4 and 10	**f.** 8 and 12

2. Find the LCM for each of the following sets of numbers.

a. 4, 8, and 12	**b.** 3, 6, and 12	**c.** 4, 8, and 10
d. 3, 6, and 9	**e.** 4, 6, and 12	**f.** 2, 6, and 10

3. You can buy wieners in packages of 10 and buns in packages of 12. If you want to have an equal number of wieners and buns, what is the least number of packages of each that you should buy?



Check your answers by turning to the Appendix.

Enrichment

Do one or more of the following questions. If you play one of the games, try to develop strategies so that you can win each time.

1. Play the game "Multiple Tick-Tack-Toe" with a friend. You will need one die, eight disks of one colour, and eight disks of another colour. Use the game board provided in the Appendix.

How to Play "Multiple Tick-Tack-Toe"[1]

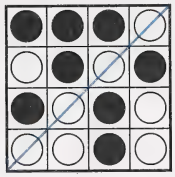
Step 1: Players roll the die to see who goes first. The player who rolls the higher number goes first.

Step 2: In turn, each player throws the die and covers a multiple of that number on the game board.

Example

If is rolled, the player can cover one of the following numbers: 6, 12, 18, 24, or 30.

Step 3: The game continues until one player is able to place four disks in a row horizontally, vertically, or diagonally.



[1] National Council of Teachers of Mathematics for the game from the *Arithmetic Teacher*, Reston, Virginia.

If you have access to a Macintosh computer and the program *Number Munchers*™ (MECC), do question 2.

2. Play the game "Multiples" from the program *Number Munchers*™.

3. Play "The LCM Game" with a friend. You will need two dice, eight disks of one colour, and eight disks of another colour. Use the game board provided in the Appendix.

How to Play "The LCM Game"[1]

Step 1: Roll the dice. The player with the highest total begins.

Step 2: In turn, each player throws the dice and by using the numbers on the top faces determines the least common multiple for those numbers.

Example

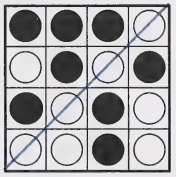
The LCM is 12 since 12 is the least number that both 6 and 4 divide into exactly.

Step 3: The number found to be the LCM is covered with a disk by the player.

Step 4: If a player rolls a combination that is covered by the opponent, the player removes that disk and replaces it with a disk of his or her own colour.

Step 5: If the player rolls a combination that is covered by a disk of his or her own colour, the turn is lost.

Step 6: The game continues until one player is able to place four disks in a row horizontally, vertically, or diagonally.



[1] National Council of Teachers of Mathematics for the game from the *Arithmetic Teacher*, Reston, Virginia.

4. The members of a marching band tried to arrange themselves in rows of 2, 3, 4, or 5 with exactly the same number of people in each row, but each time there was one person left over. What is the least number of members that the marching band could have had?

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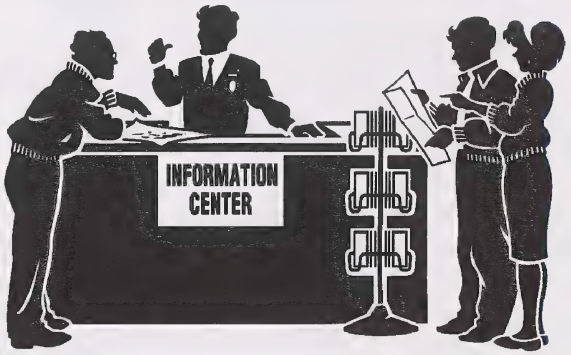
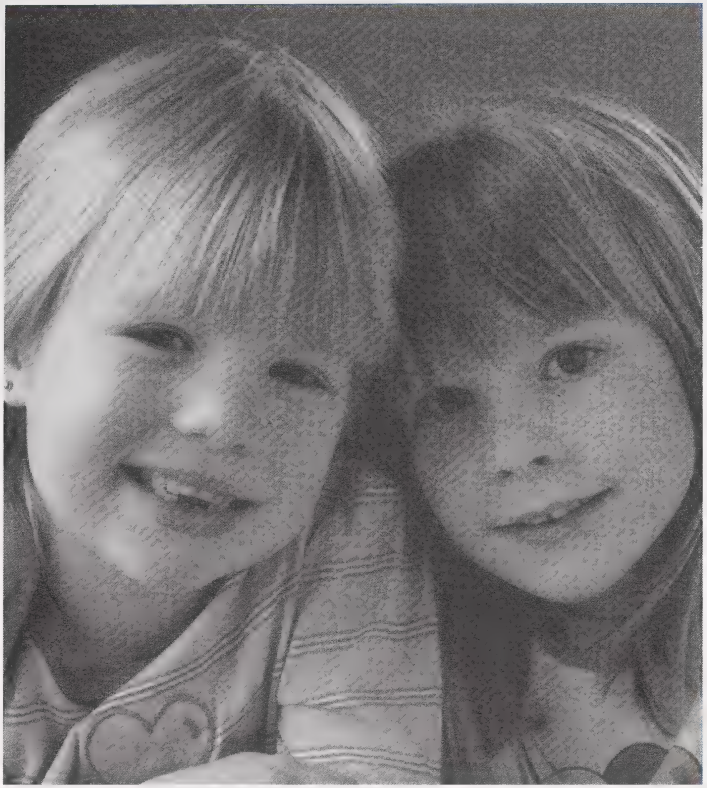
5. An information centre is showing two tourism videos. The first video is 36 minutes long. The second video is 45 minutes long. If each video begins at 8:45 A.M. and runs continuously until 5:45 P.M., at what times will both start together again?



Check your answers by turning to the Appendix.

Conclusion



In this section you explored the word *multiple* as it is used in mathematics. A multiple of a number is found by multiplying the number by another whole number or by adding the number repeatedly.

You listed the multiples of a number, found the least common multiple of two or more numbers, and solved problems by applying your knowledge of least common multiples.

If you completed the Extra Help, you used blocks to model the multiples of a number and the least common multiple of two or more numbers. You also used multiple bars to assist you in finding the least common multiple of two or more numbers. If you completed the Enrichment, you played games involving multiples or solved problems involving multiples.

The word *multiple* has a different meaning outside mathematics. Multiple births are rare in humans. Leanne and Jessica are fraternal twins. Do you know any twins, triplets, quadruplets, or quintuplets?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 1.

Section 2: Factors

Have you ever wondered why green leaves turn yellow, orange, or red in the fall? Different factors are responsible for the colour changes. One factor is the decrease in light. Another factor is the decrease in temperature.

In mathematics, as you will discover, the word *factor* has a specialized meaning.

In this section you will use division and divisibility rules to decide if a number is a factor of another number. You will list the factors of a number and you will also find the greatest common factor of two or more numbers. You will also write the product of identical factors in a compact way by using powers.

Activity 1: Deciding if a Number Is a Factor of Another Number

There are many opposite actions in life. For example, opening an umbrella is the opposite of closing an umbrella.

WESTFILE INC.

There are many opposite operations in mathematics. **Multiplying** and **dividing** are opposite operations.

The process of finding the total number of elements in a given number of groups where each group has the same number of elements is called multiplying. The process of finding the number of elements in a given number of groups is called dividing.

Example 1

To solve the multiplication problem 3×4 ask yourself, "How many elements are there in three groups of four?"

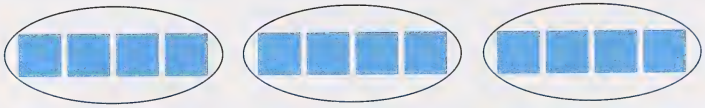
Of course, there are 12 elements.

So, $3 \times 4 = 12$.

multiplier product

Example 2

To solve the division problem $12 \div 3$ ask yourself, "In 12, there are three groups of how many?"

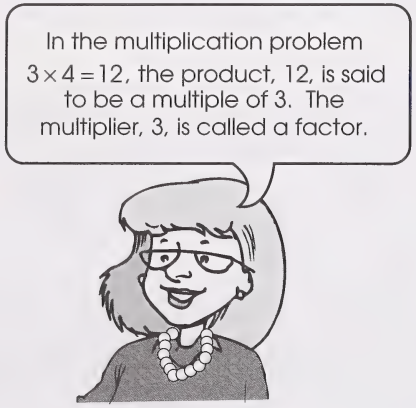


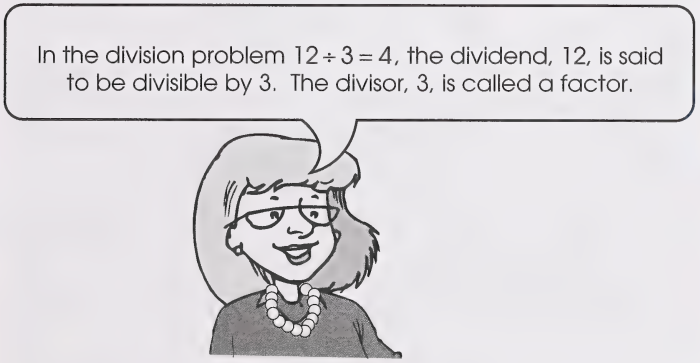
Of course, each of the three groups has four elements.

So, $12 \div 3 = 4$.

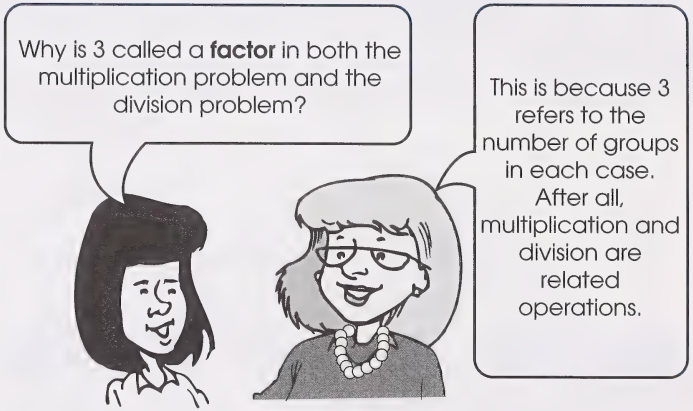
dividend divisor quotient

Several mathematical terms are used when discussing multiplication and division.



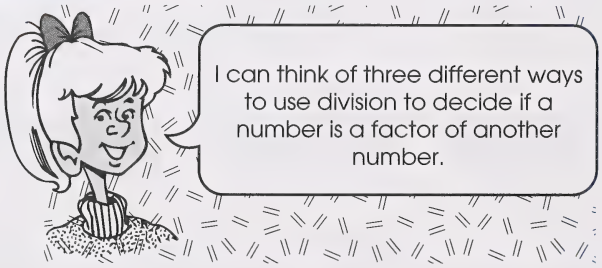


Notice that the number 3 is called a **factor** in both the multiplication problem and the division problem.


Why is 3 called a **factor** in both the multiplication problem and the division problem?
This is because 3 refers to the number of groups in each case. After all, multiplication and division are related operations.

Because of the relationship between multiplication and division, you can use division to decide if a number is a factor of another number. When the **remainder** is zero, the divisor is a factor of the dividend.

The remainder is the amount left over after dividing.



Example 3

Is 4 a factor of 252?

Solution

Method 1: Using Long Division

Use division to decide if 4 is a factor of 252.

$$\begin{array}{r} 63 \\ 4\overline{)252} \\ \underline{24} \\ 12 \\ \underline{12} \\ 0 \end{array} \longleftarrow \text{remainder}$$

Because the remainder is zero, 4 is a factor of 252.

Method 2: Using Short Division

Use division to decide if 4 is a factor of 252.

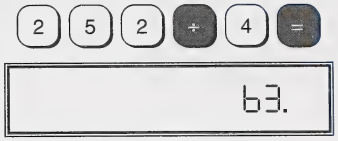
$$\begin{array}{r} 6\ 3\ \text{R0} \\ 4\overline{)25_{1}2} \end{array}$$

In short division, the multiplication and subtraction steps are done mentally.

Because the remainder is zero, 4 is a factor of 252.

Method 3: Using a Calculator

Use division to decide if 4 is a factor of 252.



Because the quotient is a whole number, 4 is a factor of 252.

Example 4

There are 676 students in Cobequid Junior High School. The principal wishes to group the students into teams for intramural sports. Each team must have the same number of students. Can the teams have exactly 9 students each?

Solution

Method 1: Using Long Division

Use division to decide if 9 is a factor of 676.

$$
\begin{array}{r}
75 \\
9\overline{)676} \\
\underline{63} \\
46 \\
\underline{45} \\
1
\end{array}
\quad \longleftarrow \text{ remainder}
$$

Because the remainder is not zero, 9 is not a factor of 676.

So, the teams cannot have exactly 9 students each.

Method 2: Using Short Division

Use division to decide if 9 is a factor of 676.

$$
\begin{array}{r}
7\ 5\ \text{R}1 \\
9\overline{)67_{4}6}
\end{array}
$$

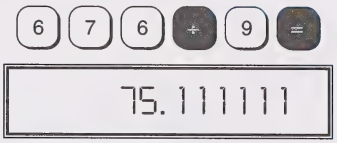
In short division, the multiplication and subtraction steps are done mentally.

Because the remainder is not zero, 9 is not a factor of 676.

So, the teams cannot have exactly 9 students each.

Method 3: Using a Calculator

Use division to decide if 9 is a factor of 676.



Because the quotient is not a whole number, 9 is not a factor of 676.

So, the teams cannot have exactly 9 students each.

1. Use long division to decide if 3 is a factor of each of the following numbers.

a. 1306 **b.** 9858 **c.** 5842

2. Use short division to decide if 4 is a factor of each of the following numbers.

a. 1163 **b.** 7232 **c.** 5018

Use a calculator to answer questions 3 and 4.

3. Is 6 a factor of each of the following numbers? Answer **yes** or **no**.

a. 1215 **b.** 8743 **c.** 5394

4. Jamaal is cutting pieces of paper 6 cm long from a roll of paper 750 cm long to make paper flowers. Can he cut the roll into 6-cm pieces without any waste?

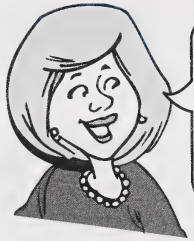
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

5. Eight students are having a chess tournament. They decided that each person would play everyone else one game of chess, and the person who won the most games would be the winner. How many games of chess will the group play?

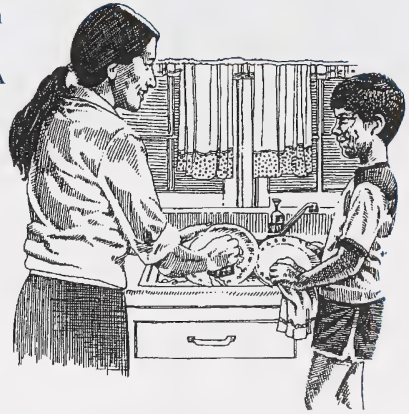
Check your answer by turning to the Appendix.



In this activity you discovered how multiplying and dividing are related. You determined whether a number is a factor of another number. You used a problem-solving strategy to solve a problem about chess.

Activity 2: Discovering and Applying Divisibility Tests

Do you do dishes by hand in your family, or do you have an automatic dishwasher? A dishwasher can save time and work.



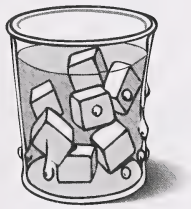
In this activity you will discover and apply several **divisibility tests** so that you can decide if a number is a factor of another number without dividing. These tests will save you time and work.

A divisibility test is a test to quickly decide if a number is able to be divided exactly by another number.

You will use **inductive reasoning** to discover divisibility tests.

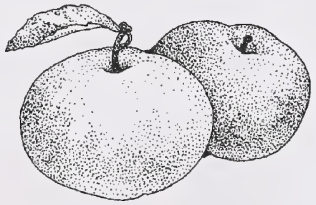
Inductive reasoning is the process of arriving at a conclusion by generalizing.

Inductive reasoning is used to discover patterns and relationships. You have been using inductive reasoning since you were a baby. For example, after touching several ice cubes, you probably concluded that ice cubes are always cold.



However, inductive reasoning must be done cautiously because it involves a certain amount of intelligent guesswork. Not all conclusions drawn by the process of induction are true.

For instance, after eating several green apples, you might conclude that all green apples are sour. This may be true for green unripe apples, but there are some varieties of apples that are green even when ripe and are not sour.



It is important that you do not jump to conclusions and that you examine many **"yes" examples** and **"no" examples** to test your conclusions.

A "yes" example is an example that shows a case that fits the definition. A "no" example is an example that shows a case that does not fit the definition.

Divisibility by 2

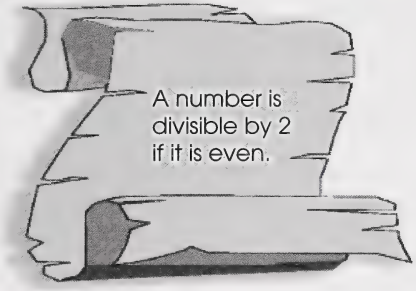
How can you decide if a number is divisible by 2? Look at the following chart. Study the "yes" and "no" examples of numbers divisible by 2. Then answer question 1.

Numbers Divisible by 2	Numbers Not Divisible by 2
1380	2151
2762	3253
3894	5615
8956	7867
7148	4389

1. What pattern do you notice from the "yes" and "no" examples of numbers divisible by 2? What can you conclude?

Check your answers by turning to the Appendix.

You have discovered the following divisibility test that will help you decide if a number is divisible by 2.

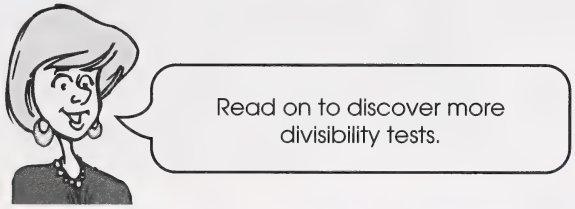


Use only this divisibility test to answer question 2.

2. Does each of the following numbers have 2 as a factor? Answer **yes** or **no**.

a. 13 582 **b.** 19 876 **c.** 45 769

Check your answers by turning to the Appendix.



Divisibility by 5 or 10

How can you decide if a number is divisible by 5 or 10?

Look at the following chart. Study the "yes" and "no" examples of numbers divisible by 5 and 10. Then answer question 3.

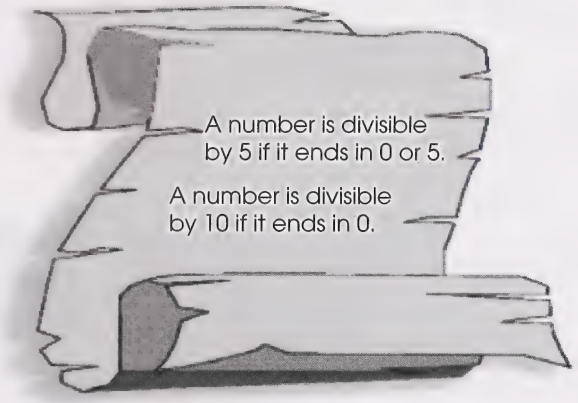
Numbers Divisible by 5 and 10	Numbers Divisible by 5, but Not 10	Numbers Divisible by Neither 5 nor 10
6180	3175	1231
9340	9285	5362
1870	5645	4173
2510	8395	5284
3450	7815	9716
9820	4925	1277
7290	2865	4088
8360	1735	3329

3. **a.** What pattern do you notice about the "yes" and "no" examples of numbers divisible by 5? What can you conclude?

b. What pattern do you notice about the "yes" and "no" examples of numbers divisible by 10? What can you conclude?

Check your answers by turning to the Appendix.

You have discovered the following divisibility tests that will help you decide if a number is divisible by 5 or 10.



Use only these divisibility tests to answer questions 4 and 5.

4. Does each of the following numbers have 5 as a factor? Answer **yes** or **no**.

a. 19 420 **b.** 56 005 **c.** 875 329

5. Does each of the following numbers have 10 as a factor? Answer **yes** or **no**.

a. 92 533 **b.** 199 345 **c.** 18 650

Check your answers by turning to the Appendix.

Divisibility by 4 and 8

How can you decide if a number is divisible by 4 or 8?

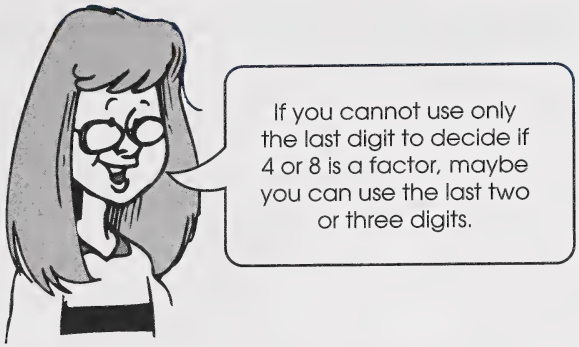
Look at the following chart. Study the "yes" and "no" examples of numbers divisible by 4 and 8. Then answer question 6.

Numbers Divisible by 4 and 8	Numbers Divisible by 4, but Not 8	Numbers Divisible by Neither 4 nor 8
78 400	63 380	36 250
63 880	30 020	46 131
57 312	45 212	90 062
12 032	88 332	75 253
49 664	27 164	51 814
27 424	16 004	86 375
92 816	72 116	71 106
85 336	15 036	93 717
29 048	54 548	27 138
34 168	95 028	88 019

6. **a.** Can you use the last digit of a number to decide if 4 or 8 is a factor? Why or why not?

b. Can you use the last digit of a number to decide if 4 or 8 is not a factor? Why or why not?

Check your answers by turning to the Appendix.



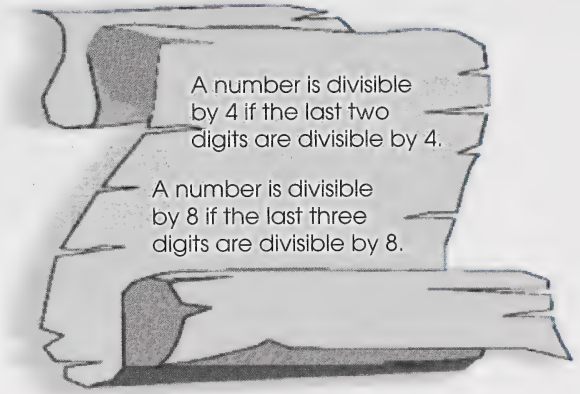
Study the "yes" and "no" examples of numbers divisible by 4 and 8. Then answer question 7.

7. a. What do you notice about the last two digits of the numbers divisible by 4 that is different from the last two digits of numbers not divisible by 4? What can you conclude?

b. What do you notice about the last three digits of numbers divisible by 8 that is different from the last three digits of numbers not divisible by 8? What can you conclude?

Check your answers by turning to the Appendix.

You have discovered the following divisibility tests to decide if a number is divisible by 4 or 8.


A number is divisible by 4 if the last two digits are divisible by 4.
A number is divisible by 8 if the last three digits are divisible by 8.

Use only these divisibility tests to answer questions 8 and 9.

8. Does each of these numbers have 4 as a factor? Answer **yes** or **no**.

a. 7168	**b.** 3354	**c.** 6528
d. 2616	**e.** 35 638	**f.** 516 082

9. Does each of these numbers have 8 as a factor? Answer **yes** or **no**.

a. 191 080	**b.** 37 206	**c.** 68 592
d. 70 034	**e.** 18 320	**f.** 365 168

Check your answers by turning to the Appendix.

Divisibility by 3, 6, and 9

How can you decide if a number is divisible by 3, 6, or 9?

Look at the following chart. Study the "yes" and "no" examples of numbers divisible by 3, 6, or 9. Then answer question 10.

Numbers Divisible by 3	Numbers Divisible by 6	Numbers Divisible by 9	Numbers Not Divisible by 3, 6, or 9
20 301	23 592	71 001	21 301
10 002	42 132	92 142	25 132
24 513	31 704	78 903	70 903
30 894	41 004	22 554	51 004
61 365	55 116	96 885	86 365
12 426	92 136	11 106	61 136
94 437	21 528	66 807	12 317
20 118	30 678	21 528	27 148
15 219	11 610	90 009	11 239
31 500	60 000	36 000	40 000

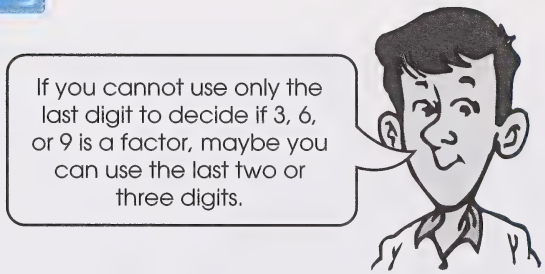
10. **a.** Can you use the last digit of a number to decide if a number is divisible by 3? Why or why not?

b. Can you use the last digit of a number to decide if a number is divisible by 6? Why or why not?

c. Can you use the last digit of a number to decide if a number is divisible by 9? Why or why not?

d. Can you use the last digit to eliminate 3, 6, or 9 as a factor? Why or why not?

Check your answers by turning to the Appendix.



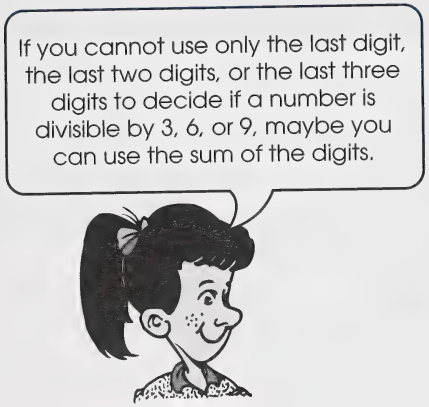
Study the "yes" and "no" examples of numbers divisible by 3, 6, or 9. Then answer question 11.

11. **a.** Can you use the last two or three digits to decide if a number is divisible by 3? Why or why not?

b. Can you use the last two or three digits to decide if a number is divisible by 6? Why or why not?

c. Can you use the last two or three digits to decide if a number is divisible by 9? Why or why not?

Check your answers by turning to the Appendix.


If you cannot use only the last digit, the last two digits, or the last three digits to decide if a number is divisible by 3, 6, or 9, maybe you can use the sum of the digits.

Study the "yes" and "no" examples of numbers divisible by 3, 6, or 9. Then answer question 12.

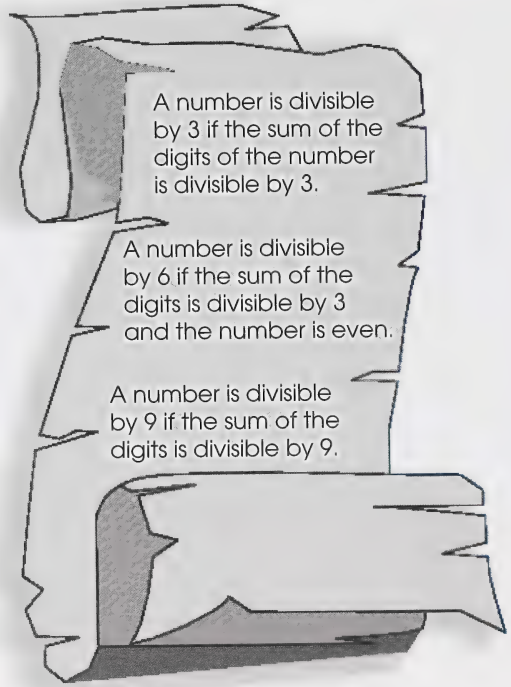
12. **a.** Is the sum of the digits of each of the numbers in the first column divisible by 3? Is the sum of the digits of any of the numbers in the last column divisible by 3? What can you conclude?

b. Which column(s) has/have numbers in which the sum of the digits is always divisible by 3 and the last digit is always even? What can you conclude?

c. In which column(s) is/are the sum of the digits divisible by 9? What can you conclude?

Check your answers by turning to the Appendix.

You have discovered the following divisibility tests to decide if a number is divisible by 3, 6, or 9.


A number is divisible by 3 if the sum of the digits of the number is divisible by 3.
A number is divisible by 6 if the sum of the digits is divisible by 3 and the number is even.
A number is divisible by 9 if the sum of the digits is divisible by 9.

Use only these divisibility tests to answer questions 13, 14, and 15.

13. Does each of these numbers have 3 as a factor? Answer **yes** or **no**.

a. 6313 **b.** 62 160 **c.** 5241

14. Does each of these numbers have 6 as a factor? Answer **yes** or **no**.

a. 28 312 **b.** 37 032 **c.** 37 293

15. Does each of these numbers have 9 as a factor? Answer **yes** or **no**.

a. 70 038 **b.** 27 099 **c.** 63 012

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

16. A computer keyboard has a problem. It beeps whenever the 3-key or the 8-key is typed. If you type the numbers from 100 to 199, how many times will the computer beep?

Check your answer by turning to the Appendix.

In this activity you discovered several divisibility tests. You used these tests to decide if one number is a factor of another. The tests are summarized in the following chart. You also solved a problem about a computer.

Divisibility Tests

Factor	Condition	Example
2	The number is even.	Since 6934 is an even number, 6934 is divisible by 2.
3	The sum of the digits is divisible by 3.	Since 8+7+5+4 is divisible by 3, 8754 is divisible by 3.
4	The last two digits are divisible by 4.	Since $24 \div 4 = 6$, 17 324 is divisible by 4.
5	The last digit is 0 or 5.	Since the last digit is 5, 17 365 is divisible by 5.
6	The number is divisible by both 2 and 3.	Since 2382 is even and 2+3+8+2 is divisible by 3, 2382 is divisible by 6.
8	The last three digits are divisible by 8.	Since $128 \div 8 = 16$, 159 128 is divisible by 8.
9	The sum of the digits is divisible by 9.	Since 8+6+4 is divisible by 9, 864 is divisible by 9.
10	The last digit is 0.	Since the last digit is 0, 320 is divisible by 10.

Activity 3: Listing the Factors of a Number

When your parents go grocery shopping, do they make a list? Making a list is helpful if you do not want to forget anything.

In this activity you will list all the factors of a number.

One way to list all the factors of a number is to begin with the number 1 and test each **consecutive number** to see if it is a factor.

Consecutive numbers are numbers in order. For example, 1, 2, 3, and 4 are consecutive numbers.

You may find it helpful to organize the numbers in a **T-table** and put a stroke through the numbers that are not factors.

A T-table is a table made in the shape of the letter "T."

Example

List all the factors of 48.

Solution

48	
1	48
2	24
3	16
4	12
~~5~~	
6	8
~~7~~	
~~8~~	~~6~~

← 5 is not a factor.

← 7 is not a factor.

← 8 and 6 have already been mentioned.

To list the factors of 48, read down the left-hand side of the T-table and read up the right-hand side.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

1. Use T-tables to list all the factors of each of the following numbers.

a. 30 **b.** 32 **c.** 45 **d.** 70

2. Mike likes to collect insects. He wants to arrange 12 butterflies in rows with the same number of butterflies in each row. What are the different possible combinations?

Hint: Find the factors of 12.

3. Complete a table like the following for the numbers 1 to 20.

Number	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5		
6		
7		
8		

4. Use your answer to question 3 to give the numbers from 1 to 20 that have exactly the following number of factors.

a. 1 **b.** 2 **c.** 3
d. 4 **e.** 5 **f.** 6

Check your answers by turning to the Appendix.

Did You Know?

A Secret Club[1]

You probably wouldn't think that anyone could lose his or her life by telling someone the answer to an arithmetic problem, but over two thousand years ago in Greece this was possible.

A group of people set up a club to study number problems. They called themselves Pythagoreans because their leader's name was Pythagoras (580–500 B.C.). They had certain ideas about numbers that they wanted to keep secret. A club member who told a secret to a nonmember could be killed.

The Pythagoreans thought numbers were like people and could be described like people. They said that **even numbers** were weak because they could all be divided exactly by 2. They said that **odd numbers** like 3, 5, and 7 were strong.

They liked the number 6 and called it **perfect** because they knew that 1, 2, 3, and 6 were the only whole numbers that divided 6 exactly. The numbers 1, 2, and 3, which are less than 6, were called **proper divisors** of 6. What made 6 perfect was that $1+2+3=6$. The number 8 is not perfect since its proper divisors, 1, 2, and 4, do not add up to 8. The number 18 is not perfect since its proper divisors, 1, 2, 3, 6, and 9, do not add up to 18.

Perfect numbers are very rare. Until 1952 only twelve perfect numbers were known. Modern computers have found more. In 1979 the twenty-seventh perfect number was found; it has 13 395 digits!

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Sometimes the Pythagoreans considered pairs of numbers. They studied the pair 220 and 284. The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110. The proper divisors of 284 are 1, 2, 4, 71, and 142. If you add each set of proper divisors, you will see why the Greeks called 220 and 284 **friendly numbers**.

Today more than six hundred pairs of friendly numbers are known, but most pairs are very large. In 1866 a sixteen-year-old Italian boy, Nicolo Paganini, discovered a pair of friendly numbers that had been missed by everybody before his time. The pair was 1184 and 1210. Can you show that these are friendly numbers?

It is thrilling to think of young Nicolo making such a wonderful discovery. Perhaps one day you will make such a discovery.

Classifying Numbers

What is a **proper divisor**?

A proper divisor of a number is a factor less than the number itself. A proper divisor is sometimes called a **proper factor**.

Example

What are the proper factors of 10?

Solution

Step 1: List the factors of 10.

10	
1	10
2	5
~~3~~	
~~4~~	
~~5~~	~~2~~

← 3 is not a factor.
← 4 is not a factor.
← 5 and 2 have already been mentioned.

The factors of 10 are 1, 2, 5, and 10.

To list the factors of 10, read down the left-hand side of the T-table and read up the right-hand side.

Step 2: Identify the factors less than 10, and list these factors.

The proper factors of 10 are 1, 2, and 5.

Numbers can be classified by the sums of their proper factors.

- If the sum of the proper factors is less than the number, the number is **deficient**.
- If the sum of the proper factors is equal to the number, the number is **perfect**.
- If the sum of the proper factors is greater than the number, the number is **abundant**.

The following chart contains examples of numbers classified by the sums of their proper factors.

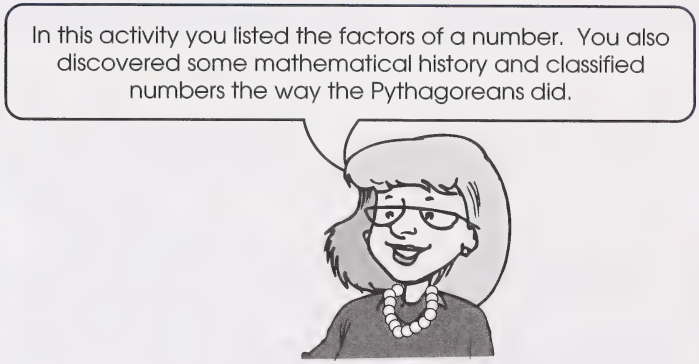
Number	Sum of Proper Factors	Kind of Number
8	1+2+4=7	deficient
6	1+2+3=6	perfect
12	1+2+3+4+6=16	abundant

Note: The number 1 has only one factor: itself. So, the number 1 has no proper factors and is not considered to be deficient, perfect, or abundant.

5. Use your answers to question 3 to complete a chart like the following for the numbers 1 to 20.

Number	Sum of Proper Factors	Kind of Number

Check your answers by turning to the Appendix.


In this activity you listed the factors of a number. You also discovered some mathematical history and classified numbers the way the Pythagoreans did.

Activity 4: Finding the Greatest Common Factor

Frank and Denny are best friends. They have many common interests. They both enjoy hockey, playing video games, and reading mysteries.

In this activity you will list the **common factors** of two or more numbers.

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A common factor of two or more numbers is a factor of each of the numbers.

You can use T-tables to find the common factors of two or more numbers.

Example 1

List the common factors of 24 and 30.

Solution

Step 1: List all the factors of 24 and all the factors of 30.

24	
1	24
2	12
3	8
4	6
~~5~~	
~~6~~	~~4~~

← 5 is not a factor.

← 6 and 4 have already been mentioned.

30	
1	30
2	15
3	10
~~4~~	
5	6
~~6~~	~~5~~

← 4 is not a factor.

← 6 and 5 have already been mentioned.

Step 2: Circle the common factors of 24 and 30. That is, circle the numbers that are in both T-tables.

24	
①	24
②	12
③	8
4	⑥
~~5~~	
~~6~~	~~4~~

30	
①	30
②	15
③	10
~~4~~	
5	⑥
~~6~~	~~5~~

The common factors of 24 and 30 are 1, 2, 3, and 6.

1. Use T-tables to list the common factors for each pair of numbers.

a. 24 and 36 **b.** 12 and 18 **c.** 30 and 75

Check your answers by turning to the Appendix.

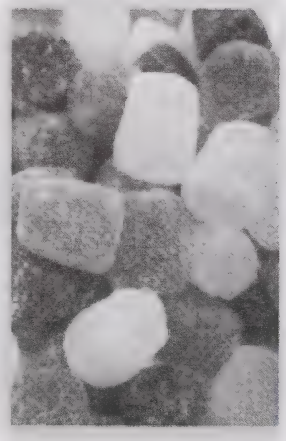
Often it is useful to find the **greatest common factor** (GCF) of two or more numbers.

The greatest common factor of two or more numbers is the greatest number in the list of common factors of the numbers.

Example 2

A grocer wants to make up gift packages that contain two kinds of candy. Each package must have the same number of candies. One type of candy comes in boxes that contain 18 pieces and the other comes in bags that contain 24 pieces. What is the greatest number of gift packages that the grocer can make from a box of one type of candy and a bag of the other type?

WESTFILE INC.



WESTFILE INC.

Solution

Step 1: List all the factors of 18 and all the factors of 24.

18	
1	18
2	9
3	6
~~4~~	
~~5~~	
~~6~~	~~3~~

24	
1	24
2	12
3	8
4	6
~~5~~	
~~6~~	~~4~~

Step 2: Circle the common factors of 18 and 24.

18	
(1)	18
(2)	9
(3)	(6)
~~4~~	
~~5~~	
~~6~~	~~3~~

24	
(1)	24
(2)	12
(3)	8
4	(6)
~~5~~	
~~6~~	~~4~~

The common factors of 18 and 24 are 1, 2, 3, and 6.

Step 3: Choose the greatest number in the list of common factors.

The greatest common factor of 18 and 24 is 6.

So, the greatest number of gift packages that the grocer can make from one box and one bag is 6.

2. Find the greatest common factor for each of these pairs of numbers.

   a. 6 and 12 **b.** 30 and 45 **c.** 42 and 56

3. What is the greatest common factor of each of these sets of numbers?

   a. 54, 81, and 63 **b.** 8, 16, and 20 **c.** 24, 42, and 48

4. Jeanine, Julie, and Joyce bought some jawbreakers at the same candy store. Jeanine bought 28¢ worth, Julie bought 72¢ worth, and Joyce bought 96¢ worth. What is the most that each jawbreaker could have cost?

Check your answers by turning to the Appendix.

Now Try This

Use the Internet to discover the nationality of each of these ancient mathematicians: Thales (624–546 B.C.), Pythagoras (585–507 B.C.), Archimedes (287–212 B.C.), and Apollonius (262–190 B.C.).

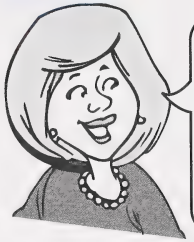
Hint: The MacTutor History of Mathematics archive on the World Wide Web has an alphabetical index of mathematicians.

Use a problem-solving strategy to answer the following question.

5. The planet Venus circles the Sun in a highly elliptical or egg-shaped orbit, ranging between 109 000 000 km from the Sun and about 107 000 000 km from the Sun.

   About how much farther from the Sun is Venus when it is at its farthest point, as compared to when it is at its closest point?

Check your answer by turning to the Appendix.

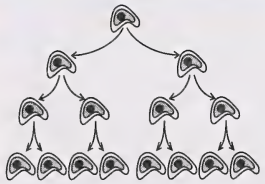


In this activity you used T-tables to find the common factors and the greatest common factor of two or more numbers. You also used the Internet to discover some mathematical history, and you solved a problem about the planet Venus.

Activity 5: Writing the Product of Identical Factors in a Compact Way

Some animals are made up of a single cell. When such a cell splits, there are then two animals. Because of cell division, a single animal can become a large colony of one-celled animals in a short time.

Study the following chart. Then answer questions 1 and 2.

Cells	Number of Splits	Number of Animals
	0	1
	1	2
	2	4
	3	8

1. What pattern do you notice in the chart?

2. Find how many animals there will be after each of the following number of splits.

a. 4 **b.** 5 **c.** 6 **d.** 7

Check your answers by turning to the Appendix.

Did you notice that the number of animals depends on the number of splits? For example, to find out how many animals there are after ten splits, write ten 2s and multiply them. This product will tell you how many animals there are after ten splits.

The number of animals after ten splits can be shown in three ways.

- in **factored form**: $2\times2\times2\times2\times2\times2\times2\times2\times2\times2$
- in **power form**: 2^{10}
- in **standard form**: 1024

A **power** is a compact way to show the product of identical factors. A power is made up of two parts —the **base** and the **exponent**. The base shows what factor is multiplied by itself. The exponent shows how many times the base is used as a factor.

For example, the base of 2^{10} is 2. So, 2 is multiplied by itself. The exponent of 2^{10} is 10. So, 2 is used as a factor 10 times.

Powers with a base of 2 are called powers of 2; 2^{10} is read as the tenth power of 2. You will learn other ways to read powers in later grades.

The standard form of a number is the usual form of the number.

3. Write each of the following in power form.

a. $3\times3\times3\times3\times3$
b. $5\times5\times5$
c. $4\times4\times4\times4\times4\times4$

4. Write each of the following in factored form.

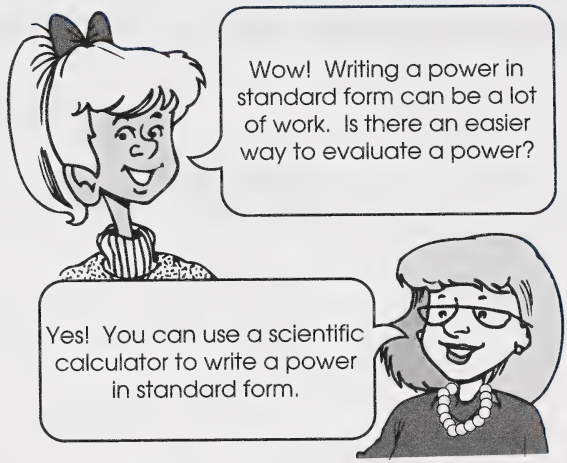
a. 5^7 **b.** 3^4 **c.** 7^2

5. Write each of the following in standard form by multiplying. The first one is done as an example.

a. $2^6 = 2\times2\times2\times2\times2\times2$
$= 4\times2\times2\times2\times2$
$= 8\times2\times2\times2$
$= 16\times2\times2$
$= 32\times2$
$= 64$

b. 3^4 **c.** 5^3 **d.** 7^4

Check your answers by turning to the Appendix.

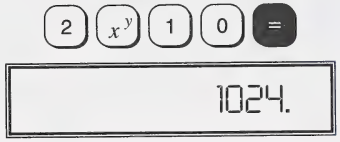

Wow! Writing a power in standard form can be a lot of work. Is there an easier way to evaluate a power?
Yes! You can use a scientific calculator to write a power in standard form.

Scientific calculators have a key which allows you to find the value of a power. The key will be labelled as x^y or y^x.

Example

Use a scientific calculator to write 2^{10} in standard form.

Solution



Use a scientific calculator to answer question 6.

6. Write each of the following powers in standard form.

a. 9^4 **b.** 12^5 **c.** 6^7 **d.** 3^8

Check your answers by turning to the Appendix.

Now Try This

Use the Internet to discover where each of these twentieth-century mathematicians was born and died: Emil Artin (1898–1962), Stefan Banach (1892–1945), Max Dehn (1878–1952), Albert Einstein (1879–1955), Kurt Godel (1906–1978), Felix Christian Klein (1849–1925), George Polys (1887–1985), Hermann Weyl (1885–1955).

Did You Know?

In this section you used the divisibility test for 2, 4, and 8. Each of these numbers is a power of 2.

$2 = 2^1$
$4 = 2^2$
$8 = 2^3$

The power forms of these numbers may help you remember the tests. Notice the relationship between the exponent and the number of digits that need to be inspected when checking for divisibility.

- A number is divisible by 2^1 if the last digit is divisible by 2.
- A number is divisible by 2^2 if the last two digits are divisible by 4.
- A number is divisible by 2^3 if the last three digits are divisible by 8.

In this activity you wrote the product of identical factors in a compact way by using powers. You evaluated powers by using a paper-and-pencil method and by using a calculator. You also used the Internet to discover facts about some twentieth-century mathematicians.

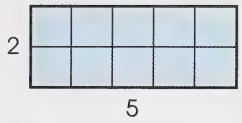
Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Models are helpful in visualizing numerical ideas.

For example, an area model can be used to show that 2 and 5 are factors of 10.



Watch the segments entitled "Factors," "Common Factors," "Greatest Common Factor," and "Powers" of the program *Number Theory* from the series *Math Moves*, and do the video activities.

You will need blocks for the activities. The introduction to the program gives you suggestions for gathering blocks.

Check your answers by viewing the video.

Enrichment

Find the page of disks for the "Factors" game in the Appendix. Photocopy the page and then cut out the disks.

Play the "Factors" game several times with a friend and try to develop a strategy so that you always win.

How to Play the "Factors" Game

Step 1: Choose the numbers with which you want to play. The numbers must be in consecutive order beginning with 1, and you must choose at least six numbers.

Lay the numbers face up on a table. Put the other numbers aside.

Step 2: Pick a number from the table. You may only pick a number which has at least one of its factors on the table. You keep the number that you choose and give your opponent all its factors.

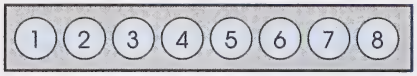
Step 3: Repeat Step 2 until you cannot pick any more numbers. Give your opponent the remaining numbers.

Step 4: Add the numbers on your side to get your score. Add the numbers on your opponent's side to get your opponent's score. The higher score wins. Who won?

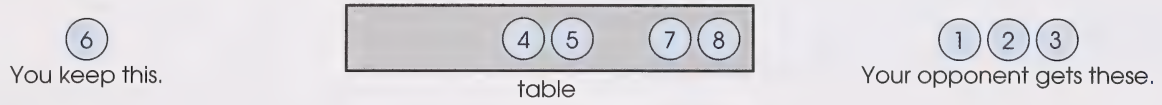
Step 5: Play another round. This time your opponent does the choosing.

Example

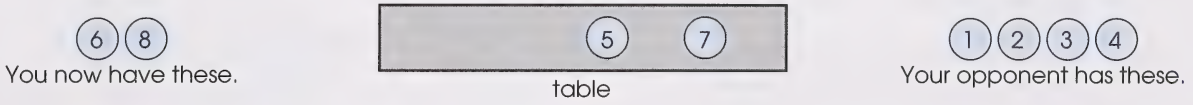
Step 1: You choose eight numbers and lay the numbers on a table.



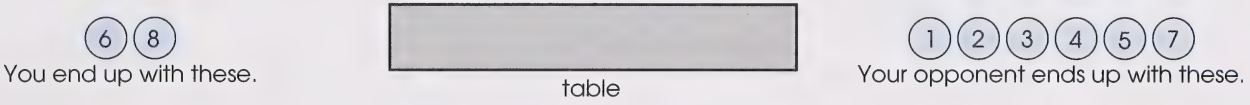
Step 2: You decide to pick the number 6 as your first number. Since the factors of 6 are 3, 2, and 1, you give your opponent these numbers.



Step 3: You then pick the number 8. (You cannot pick the 4, 5, or 7 because there are no factors of these numbers on the table.) Since 4 is a factor of 8, your opponent gets the number 4.



You cannot pick any more numbers since there are no factors of 5 or 7 on the table. Your opponent gets the numbers 5 and 7.



Step 4: You total the scores. Your score is $6+8=14$. Your opponent's score is $1+2+3+4+5+7=22$. Your opponent is the winner!

Conclusion

In this section you discovered the word *factor* has a specialized meaning in mathematics. Writing a number as a product of two or more numbers is called **factoring**. Any of the numbers used to form a product is a **factor**. For example, $30 = 1 \times 30$, $30 = 2 \times 15$, $30 = 3 \times 10$, and $30 = 5 \times 6$. So, 1, 2, 3, 5, 6, 10, 15, and 30 are factors of 30.

You decided if a number is a factor of another number by dividing or by using several divisibility tests. You used T-tables to list all the factors of a number, to find the common factors and the greatest common factors of two or more numbers, and to solve problems. You used powers to write the product of identical factors in a compact way.

The word *factor* has a different meaning outside mathematics. For example, different factors are responsible for the colour change in leaves during the fall months. (One factor is the decrease in light. Another factor is the decrease in temperature.) Here the word *factor* means any condition that helps to bring about a result.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Section 3: Prime Numbers

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Do you like prime rib roast? How are the phrases *prime rib roast*, *prime time television*, and *prime location* related? In these cases "prime" means the best part. How are words *prime minister* and *prime meridian* related? In these cases "prime" means first. How are *prime motive* and *prime of life* related? "Prime" here means most important.

In mathematics, as you will discover, the word *prime* has a specialized meaning.

In this section you will sort numbers into the categories of prime and composite and discover some properties of prime numbers. You will express numbers as a product of their prime factors. You will use the prime factorizations to find the least common multiple and the greatest common factors of two or more numbers.

Activity 1: Classifying Numbers

People seem to enjoy sorting and classifying people, animals, places, and things. For example, wind storms can be classified as gales, hurricanes, tornadoes, and so on.

In this activity you will classify numbers as **prime numbers** and **composite numbers**.

A prime number (or a prime) is a number with exactly two different factors (1 and itself.)

An example of a prime number is 5, since it has exactly two different factors. (The factors of 5 are 1 and 5.)

A composite number (or a composite) is a number with more than two different factors.

An example of a composite number is 8, since it has more than two different factors. (The factors of 8 are 1, 2, 4, and 8.)

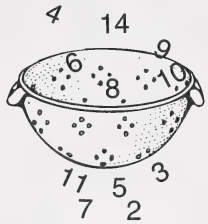
1. Is each of the following numbers prime or composite? Tell why in each case.

a. 21	**b.** 17	**c.** 39	**d.** 51
e. 73	**f.** 45	**g.** 69	**h.** 83

Check your answers by turning to the Appendix.

Eratosthenes (284–192 B.C.), an ancient Greek mathematician, invented a method called the **sieve of Eratosthenes** for separating prime numbers from other numbers.

The sieve of Eratosthenes can be used to sort the primes from as small or large a set of numbers as you like.



The following example will show you how the sieve of Eratosthenes works.

Example

Find the prime numbers from 1 to 50.

Solution

Write the numbers from 1 to 50.

Perform the following steps.

- Circle 2 and cross out all the multiples of 2.
- Circle 3 and cross out all the multiples of 3.
- Circle 5 and cross out all the multiples of 5.
- Circle 7 and cross out all the multiples of 7.
- Circle the remaining numbers.

Sieve of Eratosthenes

1	2	3	~~4~~	5	~~6~~
7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~
19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~
37	~~38~~	~~39~~	~~40~~	41	~~42~~
43	~~44~~	~~45~~	~~46~~	47	~~48~~
~~49~~	~~50~~				

The circled numbers are prime numbers. So, the prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

The crossed-out numbers are composite numbers. So, the composite numbers less than or equal to 50 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, and 50.

The number 1 is neither a prime number nor a composite number.

Use the list of prime numbers and the list of composite numbers in the example to answer questions 2 to 8.

2. **a.** Find the first two consecutive numbers between 1 and 50 that are prime numbers.

b. Find the first string of three consecutive numbers between 1 and 50 that are composite numbers.

c. Find the first string of five consecutive numbers between 1 and 50 that are composite numbers.

3. List the **twin primes** between 1 and 50.

A twin prime is a pair of prime numbers that has a difference of 2.

4. What is the sum of the first four prime numbers? Is this sum a prime number?

5. Find four pairs of prime numbers where each pair has a sum of 50.

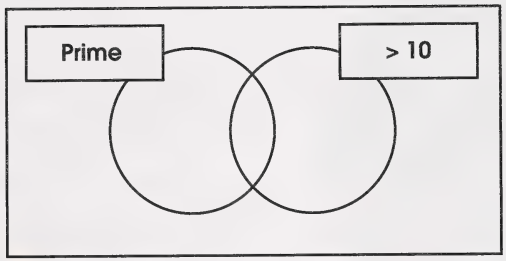
6. Christian Goldbach (1690–1764), a Russian mathematician, theorized that every even natural number greater than 4 can be expressed as the sum of two odd primes. Test his theory using each of the following numbers.

a. 6	**b.** 8	**c.** 10	**d.** 12
e. 14	**f.** 16	**g.** 18	**h.** 20

7. Christian Goldbach also theorized that every even or odd natural number greater than 7 is equal to the sum of three primes. Test his theory using each of the following numbers.

a. 8	**b.** 9	**c.** 10	**d.** 11
e. 12	**f.** 13	**g.** 14	**h.** 15

8. Sort the numbers from 1 to 20 using the given Venn diagram.

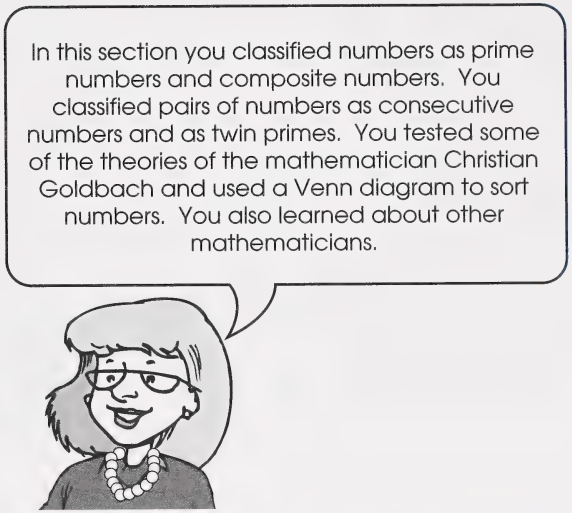


Check your answers by turning to the Appendix.

Did You Know?

Prime numbers have intrigued many mathematicians. The German mathematician Carl Fredrich Gauss (1777–1855) told a friend that whenever he had a spare 15 minutes he would spend it counting the primes in a **chiliad** (a range of 1000 numbers). By the end of Gauss' lifetime, it is estimated that he had counted all the prime numbers between 1 and 3 000 000.

An Austrian astronomer named J.P. Kulik (1773–1863) used the sieve of Eratosthenes to find all the prime numbers between 1 and 100 000 000. This activity took him twenty years to complete. Unfortunately, the library to which Kulik gave his manuscript lost the section that contained the prime numbers between 12 642 000 and 22 852 800.


In this section you classified numbers as prime numbers and composite numbers. You classified pairs of numbers as consecutive numbers and as twin primes. You tested some of the theories of the mathematician Christian Goldbach and used a Venn diagram to sort numbers. You also learned about other mathematicians.

Activity 2: Finding the Prime Factorization of a Number

Metals either occur naturally in the earth or are produced by combining two or more basic metals. For example, gold occurs naturally. Pewter is formed by combining the basic metals tin and copper.

Whole numbers greater than one are either prime numbers or are the product of two or more **prime factors**.

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A prime factor is a factor that is a prime number.

You can use T-tables to find the prime factors of a number.

Example 1

What are the prime factors of 18?

Solution

Step 1: List the factors of 18.

18	
1	18
2	9
3	6
~~4~~	
~~5~~	
~~6~~	~~3~~

← 4 is not a factor of 18.
← 5 is not a factor of 18.
← 6 and 3 have already been mentioned.

Step 2: Circle the prime factors of 18.

18	
1	18
(2)	9
(3)	6
~~4~~	
~~5~~	
~~6~~	~~3~~

The prime factors of 18 are 2 and 3.

1. List the prime factors of each of the following numbers.

a. 15 **b.** 10 **c.** 16 **d.** 24

Any number can be written in the form of a **prime factorization**.

A prime factorization is an expression of a number written as the product of its prime factors.

One way to find the prime factorization is by making a **factor tree**.

A factor tree is a diagram used to find the prime factorization of a number. In a factor tree any number that can be factored is represented on the next lower line of the tree as a product of two factors. The tree is continued until all the numbers in the bottom line are prime factors.

Note: The factors in the factor tree can be arranged in different ways; however, the resulting prime factorization is always the same.

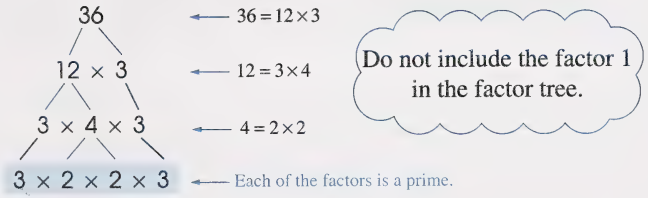
It is customary to arrange the prime factors from least to greatest.

Example 2

Use factor trees to express 36 as the product of prime factors.

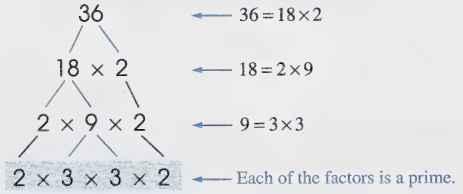
Solution

Tree 1



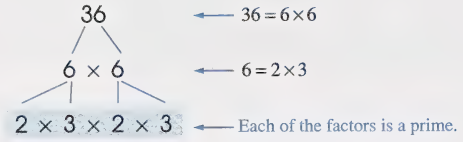
So, $36 = 2 \times 2 \times 3 \times 3$.

Tree 2



So, $36 = 2 \times 2 \times 3 \times 3$.

Tree 3



So, $36 = 2 \times 2 \times 3 \times 3$.

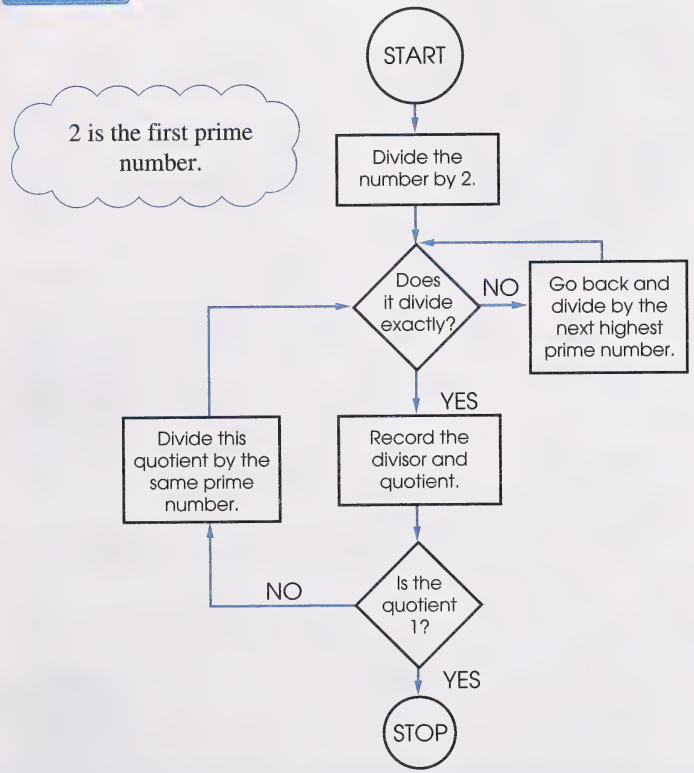
2. To find the prime factorization for each of these numbers, make a factor tree.

a. 38 **b.** 42 **c.** 45

Check your answers by turning to the Appendix.

Another way to find the prime factorization of a number is to use repeated division. The steps listed in the following **flow chart** outline this method.

A flow chart is a method for listing instructions.



Example 3

Use repeated division to express 48 as the product of prime factors.

Solution

Follow the steps in the flow chart.

$$
\begin{array}{r|l}
2 & 48 \quad \leftarrow \text{48 is divided by 2.} \\
2 & 24 \quad \leftarrow \text{24 is divided by 2.} \\
2 & 12 \quad \leftarrow \text{12 is divided by 2.} \\
2 & 6 \quad \leftarrow \text{6 is divided by 2.} \\
3 & 3 \quad \leftarrow \text{3 is divided by 3.} \\
 & 1 \quad \leftarrow \text{The quotient is 1. Stop.}
\end{array}
$$

The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3$.

3. Use repeated division to find the prime factorization for each of these numbers.

a. 100 **b.** 108 **c.** 110 **d.** 99

Check your answers by turning to the Appendix.

Now Try This

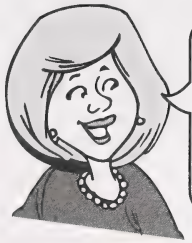
Use a problem-solving strategy to answer the following question.

4. Five jars are in a row. Each jar contains one penny more than the jar to its left. The last jar has twice as many pennies as the first. How many pennies are there in all?

Check your answer by turning to the Appendix.

Did You Know?

Reverend Charles Dodgson (1832–1898), a well-known mathematician and logician at Oxford University in England, was also famous for his writing ability. He wrote *Alice's Adventures in Wonderland* and *Through the Looking Glass* using the pen name Lewis Carroll. Have you read either of these books?



In this activity you wrote numbers in the form of a prime factorization. You did this by using factor trees or repeated division. You also solved a problem about pennies and discovered that Lewis Carroll was a mathematician as well as a writer.

Activity 3: Finding the Greatest Common Factor

Sometimes plans for buildings are drawn on the computer.

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At other times the plans are drawn using other mathematical tools.

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There is often more than one way to accomplish a task.

In the previous section you found the greatest common factor of two or more numbers by listing the factors of each number, circling the common factors, and choosing the greatest of these common factors.

You can also find the GCF of two or more numbers by writing the prime factorizations of each of the numbers, arranging the prime factorizations so that the common prime factors are aligned in columns, taking one factor from each complete column, and multiplying these factors.

Example 1

Helmut paid $78 for several doorknob sets. Eddie bought several doorknob sets for $117. If all the doorknob sets were the same price, what is the most that each doorknob set could have cost?

Solution

Step 1: Use factor trees to write the prime factorization of each number.

$117 = 3 \times 3 \times 13$

$78 = 2 \times 3 \times 13$

Step 2: Arrange the prime factorization of each number so that the common prime factors are aligned in columns.

$$\begin{aligned} 117 &= 3 \times 3 \times 13 \\ 78 &= 2 \times 3 \times 13 \end{aligned}$$

The common prime factors are 3 and 13.

Step 3: To find the GCF, take one factor from each **complete** column and multiply these factors.

$$\begin{aligned} GCF &= 3 \times 13 \\ &= 39 \end{aligned}$$

So, the most each doorknob set could cost is $39.

1. Use factor trees to find the GCF of each set of numbers.

a. 24 and 36 **b.** 12 and 18 **c.** 20, 30, and 75

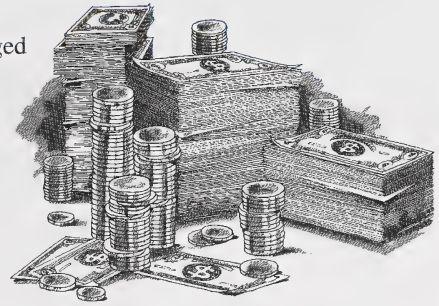
Check your answers by turning to the Appendix.

In question 1 you used factor trees to find the greatest common factor of two or more numbers.

You can also find the GCF of two or more numbers by using repeated division.

Example 2

A set of 36 nickels and 48 quarters is to be arranged in piles. If each pile must contain the same number of coins and all piles must contain only nickels or only quarters, what is the greatest number of coins that each of the piles can have?



Solution

Step 1: Use repeated division to write the prime factorization of each number.

2	36
2	18
3	9
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

Step 2: Arrange the prime factorization of each number so that the common prime factors are aligned in columns.

$$36 = 2 \times 2 \times 3 \times 3$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Step 3: To find the GCF, take one factor from each **complete** column, and multiply these common factors.

$$\text{GCF} = 2 \times 2 \times 3$$
$$= 12$$

So, the greatest number of coins that each pile can have is 12.

2. Use repeated division to find the GCF of each set of numbers.

a. 48 and 64 **b.** 54 and 63 **c.** 12, 18, and 30

Check your answers by turning to the Appendix.

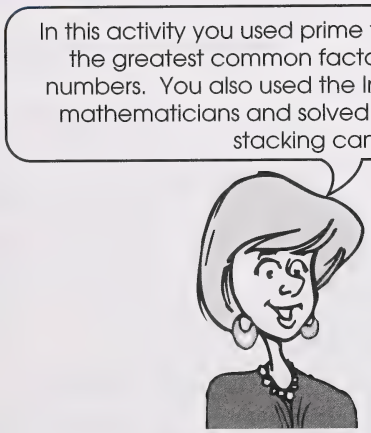
Now Try This

Use the Internet to list five famous mathematicians born in the sixteenth century. **Hint:** The MacTutor History of Mathematics archive has a chronological index.

Use a problem-solving strategy to answer the following question.

3. Kris is stacking cans for a display in the grocery store window. She wants to make the stack look like a triangle. She plans to start with a row of cans at the bottom, and put one less can in each row as she goes up, ending with one can at the very top. If she has 36 cans to stack in the display, how many cans should Kris begin with on the bottom row?

Check your answer by turning to the Appendix.


In this activity you used prime factorization to find the greatest common factor of two or more numbers. You also used the Internet to research mathematicians and solved a problem about stacking cans.

Activity 4: Finding the Least Common Multiple

Hans and Vern are chefs. Hans likes to fry hamburgers. Vern likes to barbecue them.

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There is often more than one way to accomplish a task.

In Section 1 you found the least common multiple of two or more numbers by listing the multiples, circling the common multiples, and choosing the least of the common multiples.

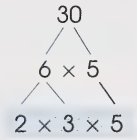
You can also find the LCM of two or more numbers by writing the prime factorization of the numbers, arranging the prime factorizations so that the common prime factors are aligned in columns, taking a factor from each column, and multiplying these factors.

Example 1

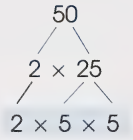
A surveyor begins putting red and blue pegs into the ground starting at a particular tree. He puts a red peg every 30 m and a blue peg every 50 m. How far from the tree will he be when he puts in both a red peg and a blue peg?

Solution

Step 1: Write the prime factorization of each number.



The prime factorization of $30 = 2 \times 3 \times 5$.



The prime factorization of $50 = 2 \times 5 \times 5$.

Step 2: Arrange the prime factorizations so that the common prime factors are aligned in columns.

$30 = 2 \times 3 \times 5$
$50 = 2 \times 5 \times 5$

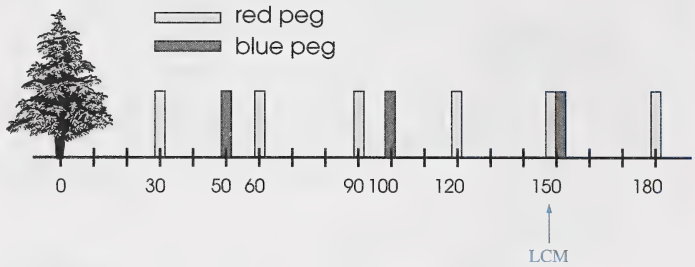
Step 3: To find the LCM, take one prime factor from each column and multiply.

$$\text{LCM} = 2 \times 3 \times 5 \times 5$$
$$= 150$$

The LCM is 150.

So, the surveyor will be 150 m from the tree when he puts in both a red peg and a blue peg.

Note: This diagram may help you visualize the situation.



1. Use factor trees to find the LCM for each set of numbers.

a. 24 and 36 **b.** 12 and 18 **c.** 20, 30, and 75

Check your answers by turning to the Appendix.

In question 1 you used factor trees to find the LCM of two or more numbers. You may also use repeated division.

Example 2

Mrs. Rasminsky found that her flowers could be arranged in rows of exactly 10 or 12 with none left over. What is the least number of flowers in the garden?

Solution

Step 1: Write the prime factorization of each number.

$$\begin{array}{r|l} 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

The prime factorization of $10 = 2 \times 5$.

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

The prime factorization of $12 = 2 \times 2 \times 3$.

Step 2: Arrange the prime factorizations so that the common factors are aligned in columns.

$$10 = 2 \quad \times 5$$
$$12 = 2 \times 2 \times 3$$

Step 3: To find the LCM, take one prime factor from each column and multiply.

$$\text{LCM} = 2 \times 2 \times 3 \times 5$$
$$= 60$$

The LCM is 60.

So, the garden has at least 60 flowers.

2. Use repeated division to find the LCM for each set of numbers.

a. 54 and 81 **b.** 48 and 80 **c.** 12, 18, and 30

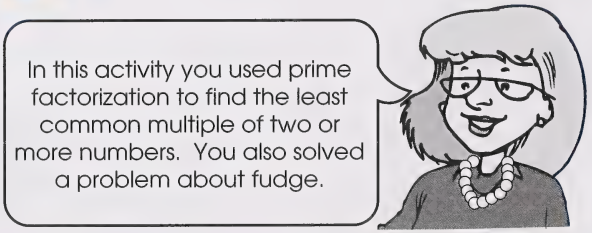
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

3. A piece of fudge costs \$0.30, \$0.40, or \$0.60, depending on the type you buy. Yves bought the same number of \$0.40 fudge as \$0.60 fudge. He also bought some \$0.30 fudge. If he paid \$6.90 for 15 pieces of fudge, how many pieces of \$0.30 fudge did he buy?

Check your answer by turning to the Appendix.


In this activity you used prime factorization to find the least common multiple of two or more numbers. You also solved a problem about fudge.

Follow-up Activities

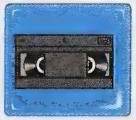
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Prime Numbers

In this section you determined whether a number was a prime number or a composite number.

Models are useful to help you visualize mathematical concepts.



View the segment entitled "Prime and Composite Numbers" of the program *Number Theory* from the series *Math Moves* and do the video activities.

You will need blocks for the activities. The introduction to the video gives you ideas for gathering the blocks.

Check your answers by viewing the solutions in the video.

Games are also helpful in developing skills.

Do either Part A **or** Part B.

Part A

If you have access to a Macintosh computer and the computer game *Number Munchers*™ (MECC), play the game "Primes."

Part B

Play the game "Place Roll" with a friend. You will need a pair of dice, several disks or bingo chips of two colours and a game board. The game board is found in the Appendix.

How to Play "Place Roll"[1]

Step 1: Each player uses disks of one colour. Choose the starting player by rolling the dice. The player with the higher number goes first.

Step 2: The starting player rolls the dice and forms a number with the top two faces of the dice. The larger number goes in the tens position and the smaller number goes in the ones position.

[1] National Council of Teachers of Mathematics for game from the *Arithmetic Teacher*, September, 1987, Reston, Virginia.

Example: = 43

Step 3: The player then puts one disk, and only one, in any box on the playing board that contains a factor of that number.

Example: = 32

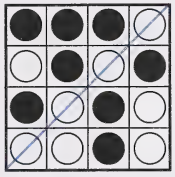
The player can cover 2, 4, 8, or 16.

Step 4: If the player rolls a double such as 33, 22, or 11, the turn is lost.

Step 5: If the player rolls a prime such as 61, 53, 43, 41, or 31, he or she can place a disk on any "prime" box.

Step 6: If a box is already occupied, a player cannot use it.

Step 7: The winner is the first player to cover four boxes in a row.



GCF and LCM

In this section you used prime factorization to find the greatest common factor (GCF) and least common multiple (LCM) of two or more numbers. Be careful not to confuse the two terms.

To find either the GCF or the LCM of two or more numbers, first arrange the prime factorizations of the numbers in columns so the common prime factors are aligned. Next, take one factor from each **complete** column and multiply to find the GCF. Take one factor from **each** column and multiply to find the LCM.

Example

Find the greatest common factor and the least common multiple of 24 and 30.

Solution

Method 1: Using Repeated Division

Step 1: Write the prime factorization of each number.

2 | 24	2 | 30
2 | 12	3 | 15
2 | 6	5 | 5
3 | 3	1
1	

$24 = 2 \times 2 \times 2 \times 3$

$30 = 2 \times 3 \times 5$

Step 2: Arrange the prime factorizations so that the common prime factors are aligned in columns.

$$24 = 2 \times 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$

Step 3: To find the GCF, take one factor from each **complete** column and multiply.

$$GCF = 2 \times 3$$
$$= 6$$

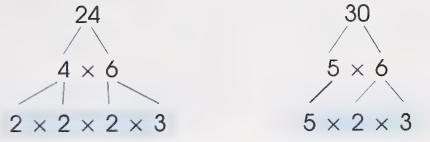
Step 4: To find the LCM, take one factor from **each** column and multiply.

$$LCM = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 120$$

The GCF of 24 and 30 is 6; the LCM is 120.

Method 2: Using Factor Trees

Step 1: Write the prime factorization of each number.



$24 = 2 \times 2 \times 2 \times 3$

$30 = 2 \times 3 \times 5$

Step 2: Arrange the prime factorizations so that the common prime factors are aligned in columns.

$$24 = 2 \times 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$

Step 3: To find the GCF, take one factor from each **complete** column and multiply.

$$GCF = 2 \times 3$$
$$= 6$$

Step 4: To find the LCM, take one prime factor from **each** column and multiply.

$$LCM = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 120$$

The GCF of 24 and 30 is 6; the LCM is 120.

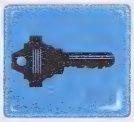
1. Find the GCF and the LCM for 18 and 36.

2. Find the GCF and the LCM for 44 and 52.

Check your answers by turning to the Appendix.

Enrichment

In this section you explored prime numbers. Do you know what **relatively prime numbers** are?



Two or more numbers with a greatest common factor of 1 are said to be relatively prime.

A "yes" example of two relatively prime numbers is 10 and 21 because the greatest common factor of 10 and 21 is 1.

A "no" example of two relatively prime numbers is 10 and 15 because the greatest common factor of 10 and 15 is 5.

1. Are the following sets of numbers relatively prime? Answer **yes** or **no** and tell why or why not.

a. 21 and 36	**b.** 87 and 53	**c.** 45 and 38
d. 21 and 93	**e.** 74, 26, and 21	**f.** 8, 21, and 36

2. What are the lowest three numbers that have the greatest common factor of 48?

Check your answers by turning to the Appendix.

Did You Know?

Aryabhata[1]

The ancient Hindus loved to do number puzzles. A mathematician named Aryabhata, who lived in India during the sixth century A.D., liked this kind of puzzle: If 4 is added to a certain number, the result divided by 2, that result multiplied by 5, and then 6 subtracted from that result, the answer is 29. What is the number?

Aryabhata used the method of inversion to solve this kind of problem. This means he worked backwards and did the inverse, or opposite, steps. Adding and subtracting are inverse steps, and multiplying and dividing are inverse steps.

Look back at the puzzle. Aryabhata started with 29 and added 6 to get 35. Then he divided by 5 to get 7. Then he multiplied by 2 to get 14. Then he subtracted 4 to get 10. The number is 10.

Use Aryabhata's method in each of the following problems.

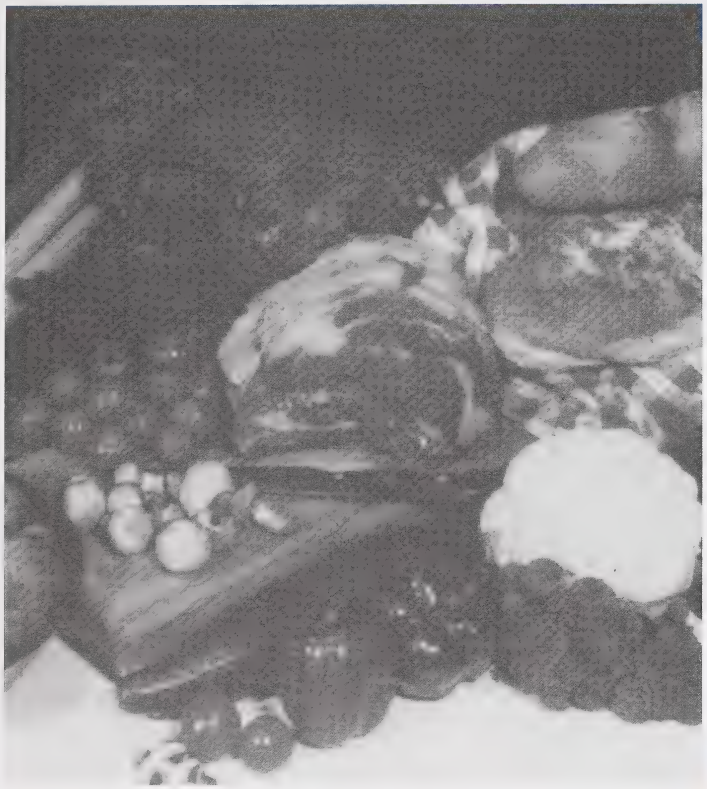
3. If a number is divided by 6, the result multiplied by 5, and 8 added to that result, the answer is 23. Find the number.

4. If 10 is added to a number, then 3 added to the result, that result divided by 4, that result multiplied by 7, and 2 subtracted from that result, the answer is 40. Find the number.

Check your answers by turning to the Appendix.

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Conclusion



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In mathematics, the word *prime* has a specialized meaning. A prime number is a number that has only the factors of 1 and itself.

In this section you classified numbers as prime numbers and composite numbers. You discovered that the number 1 is neither a prime number nor a composite number.

You also found consecutive prime numbers, consecutive composite numbers, and twin primes.

You investigated questions such as this: Can every natural number greater than 4 be expressed as the sum of two odd primes?

You discovered that numbers can be expressed as a product of their prime factors and that these prime factorizations are helpful in finding the least common multiple and greatest common factors of two or more numbers.

Outside mathematics, *prime* has a different meaning. In the phrases *prime rib roast*, *prime time television*, and *prime location* "prime" means the best part. In the phrases *prime minister*, *prime motive*, and *prime of life* "prime" means the most important.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 3.

Module Summary

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In Module 1 you increased your knowledge of the structure and properties of numbers. You listed the multiples and factors of a number, found the least common multiple and greatest common factor of two or more numbers, and used Venn diagrams to sort numbers. You used division and divisibility tests to decide if a number was a factor of another number. You used powers as a compact way of writing the product of identical factors. You discovered properties of prime numbers and used prime factorization to find the least common multiple and greatest common factor of two or more numbers.

Numbers are used frequently in daily life. You are probably very familiar with situations where numbers are used to represent people, places, and products.

How do you use numbers in your life?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment for Module 1.

APPENDIX

- Problem-Solving Strategies
- Glossary
- Suggested Answers
- Cut-out Learning Aids

Problem-Solving Strategies

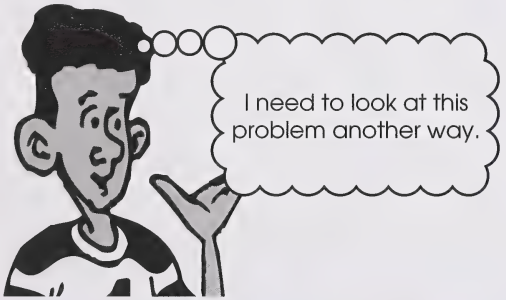
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagrams, graphs, or charts that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not being successful, it is often helpful to try to change your point of view.

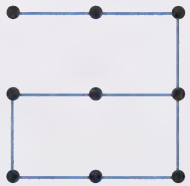


Example

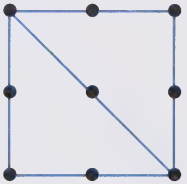
Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

Solution

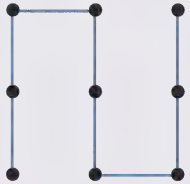
Step 1: You make several attempts at solving the problem.



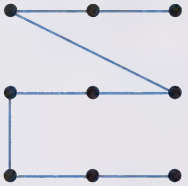
5 line segments



5 line segments



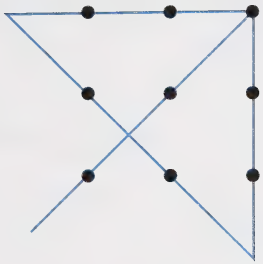
5 line segments



5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem you must change your point of view and realize that the line segments may lie outside the confines of the dots.



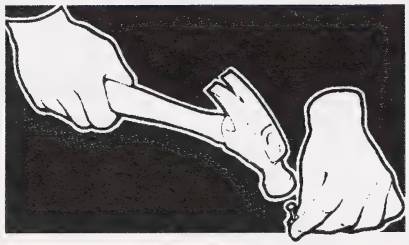
4 line segments

Using Objects

Objects can be used to help you solve a problem.

Example 1

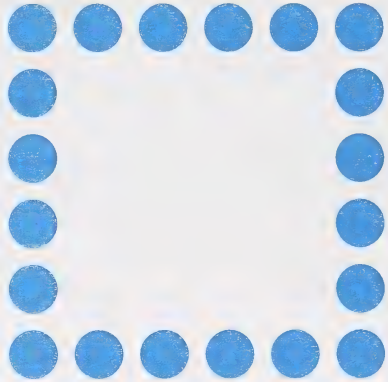
To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?



Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

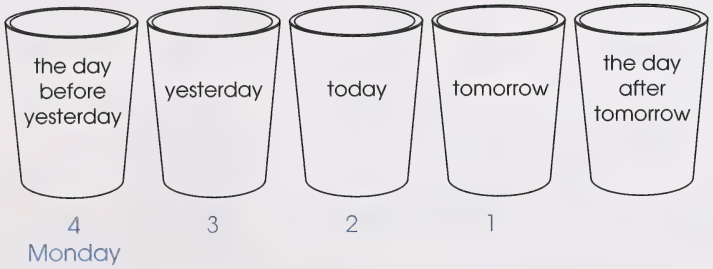
Example 2

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.



If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

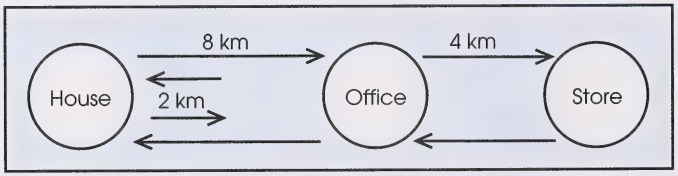
You can use sketches to solve problems.

Example 1

Chris' office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2+2+8+4+4+8=28$$

Chris drove 28 km.

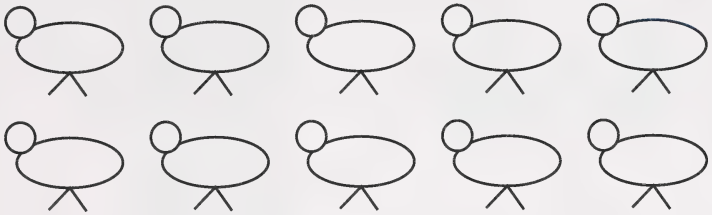
Example 2

There are ten animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

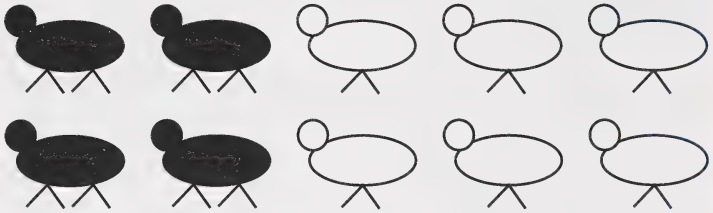
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of ten animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



There are four sheep and six chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example 1

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.

If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

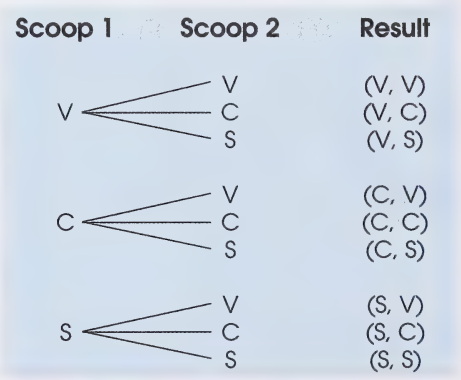
There are ten possible point totals.

Example 2

At an ice-cream parlour there are three flavours of ice cream. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate cone. Order makes a difference.

Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

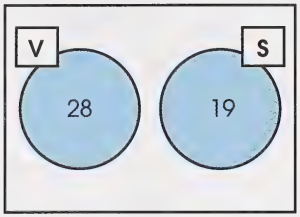
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician named John Venn (1834–1923).

Example 1

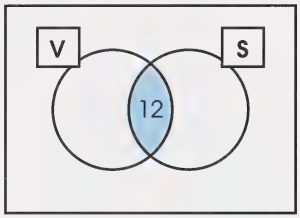
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



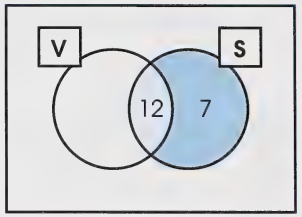
Step 2: Represent the members of both teams by the intersection of these two circles.



Step 3: Now calculate the number of girls who belong only to the swim team.

$$19 - 12 = 7$$

So, 7 girls belong only to the swim team.

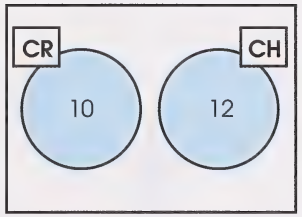


Example 2

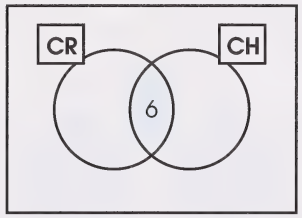
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states 10 plates were cracked and 12 plates were chipped. Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

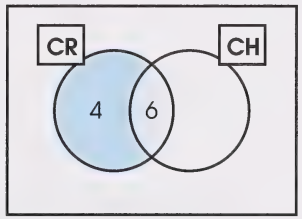


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



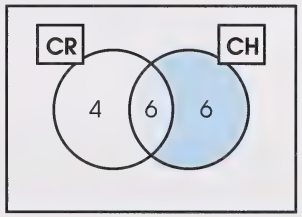
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

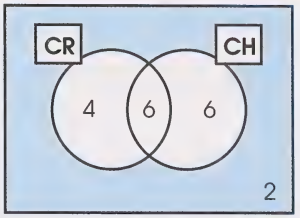


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



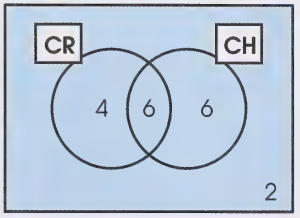
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example 3

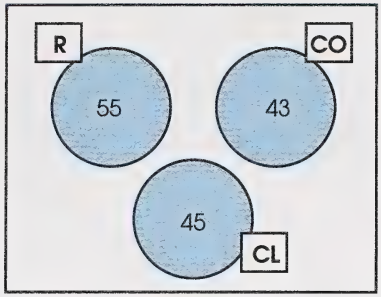
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country music and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

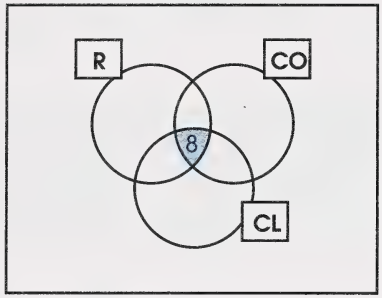
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music, 43 like country music, and 45 like classical music. Use three circles to represent these preferences.

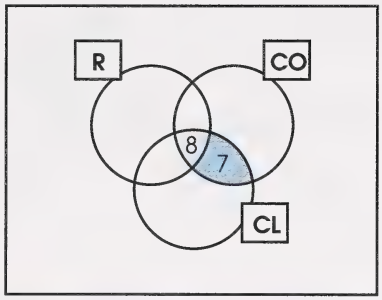


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



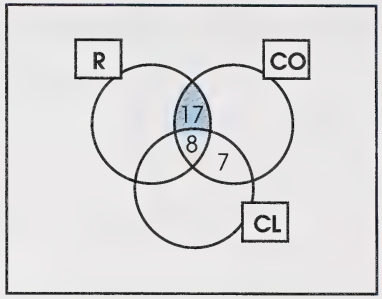
Step 3: The problem states that 15 people like both country music and classical music. Calculate the number of people who like country and classical music, but do not like rock music.

$$15-8=7$$



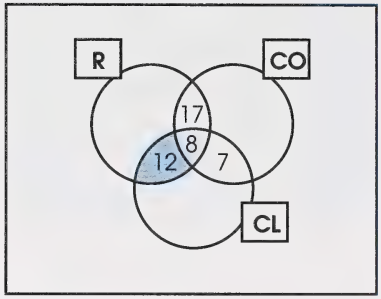
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but do not like classical music.

$$25-8=17$$



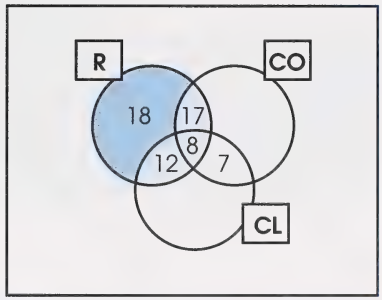
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20-8=12$$



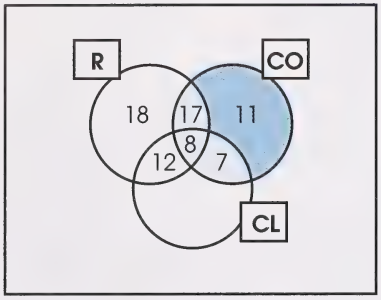
Step 6: Now that you know the number of people who like rock music and another form of music, calculate the number of people who only like rock.

$$55-(17+8+12)=18$$



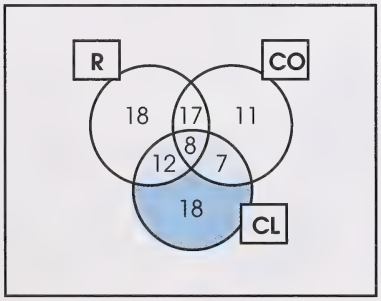
Step 7: Now that you know the number of people who like country music and another form of music, calculate the number of people who only like country music.

$$43-(17+8+7)=11$$



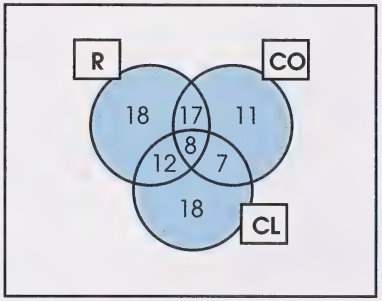
Step 8: Now that you know the number of people who like classical music and another form of music, calculate the number of people who like only classical music.

$$45-(12+8+7)=18$$



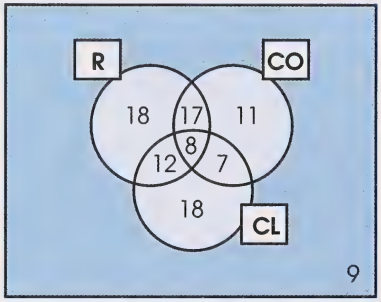
Step 9: Now you can calculate the total number of people who like rock, country, or classical music.

$$18+17+8+12+11+7+18=91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100-91=9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many customers did she assist that day and how many bags did she pack?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

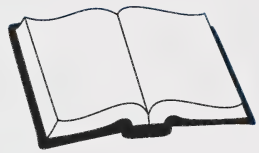
correct guess

Hannah assisted 12 customers and packed 7 bags that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example



Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.

Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess, check your guess, and revise if necessary.

It is helpful to organize your guesses in a table.

Left-hand page number	50	52	54
Right-hand page number	51	53	55
Product of page numbers	2550	2756	2970

correct guess

So, the book was open to pages 54 and 55.

Method 2: Using a Computer

Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells on the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead you use the entry bar at the top of the spreadsheet.

A5 | × | √ | Product of page numbers

	A	B
1	Book Problem	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

From the problem, you know that the right-hand page number is one more than the left-hand page number and that the product of the page numbers is 2970.

Here is what you enter:

- Begin with the B3 cell and enter a guess of 50.
- Next, enter = B 3 + 1 for the B4 cell. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter = B 3 * B 4 for the B5 cell. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 | × | √ | =B3*B4

	A	B
1	Book Problem	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		

The computer performs the calculations and the results appear in the B4 and B5 cells.

Step 3: Check your initial guess and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter 54.

B3 | × | ✓ | 54

	A	B
1	Book Problem	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in the B4 and B5 cells.

Step 4: Check your initial guess and revise it if necessary. The product is 2970.

So, the book was open to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.
- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange.

Each boy will have 6 cards.

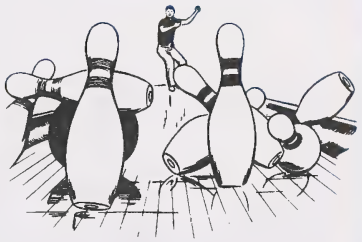
If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange.

Jon will have 8 cards and Matt will have 4 cards.

Working Backwards

In mathematics you are usually given information, and then asked to find the answer. Sometimes, however, you are given the answer and then you are asked to find a piece of information. In cases like these, you need to work backwards.

Example 1



Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What minimum score must he make in his third game to maintain his average?

Solution

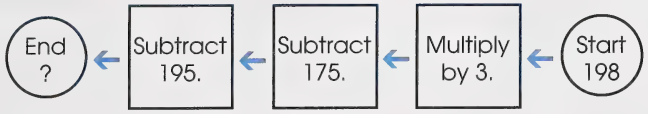
Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.

You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.

You do not know one of the three scores, so you cannot do the first step, but you can work backwards.

Step 2: Use a reverse flow chart to work backwards.

These operations undo operations in the original flow chart.



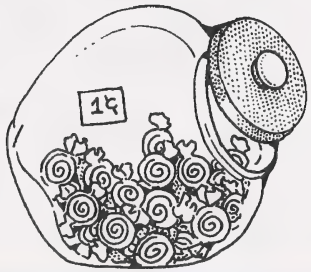
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the minimum score that Ross must get in the third game.

Ross must get at least 224 in the last game.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

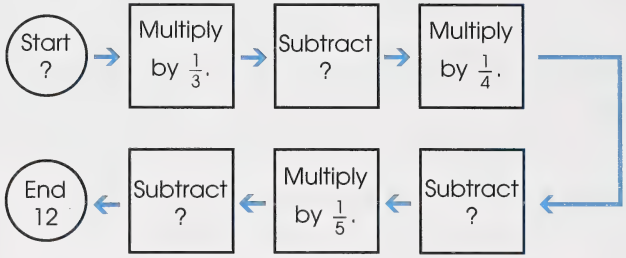
Example 2

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

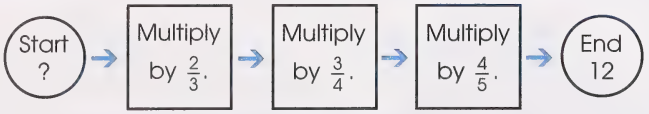


This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

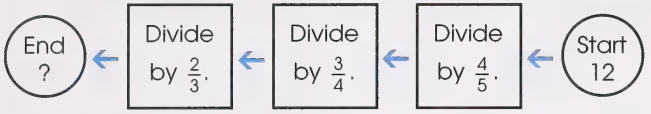
Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds. $1-\frac{1}{3}=\frac{2}{3}$
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths. $1-\frac{1}{4}=\frac{3}{4}$
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths. $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

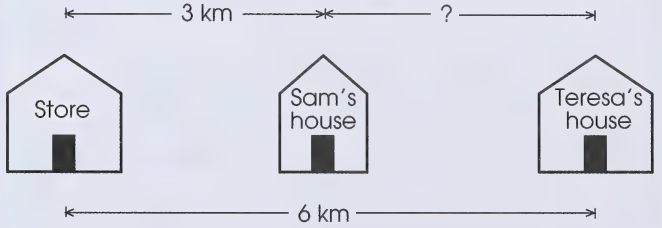
Example

The average distance from the Sun to Mars is 228 000 000 km and the average distance from the Sun to Earth is 150 000 000 km. What is the distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

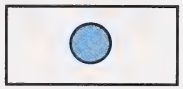
Example

There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

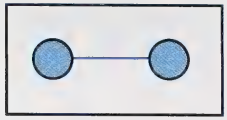
Solution

Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

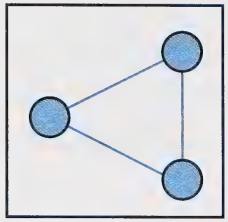
- If there is 1 student, no handshakes can be exchanged, as illustrated by the diagram at the right.



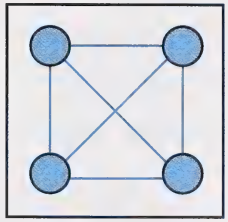
- If there are 2 students, 1 handshake can be exchanged, as illustrated by the diagram at the right.



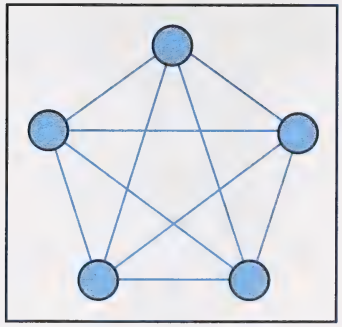
- If there are 3 students, 3 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 4 students, 6 handshakes can be exchanged, as illustrated by the diagram at the right.



- If there are 5 students, 10 handshakes can be exchanged, as illustrated by the diagram at the right.

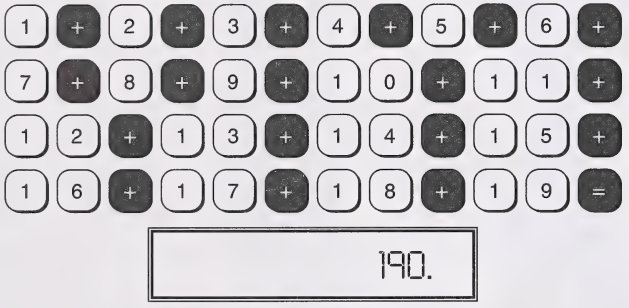


Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern.

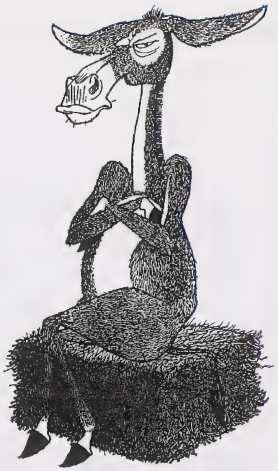


The total number of handshakes exchanged will be 190.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example



Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and coyote did not finish first. They did not beat the lion.
- The rabbit did not finish first. The coyote beat it.
- The coyote did not finish first. The ostrich was faster.
- The donkey did not finish first. It lagged behind the rabbit.
- The ostrich did not finish first. It was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now reconsider the clues:

- The rabbit did not finish second. The coyote beat it.
- The coyote did not finish second. The ostrich was faster.
- The donkey did not finish second. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now reconsider the clues:

- The rabbit did not finish third. The coyote beat it.
- The donkey did not finish third. It lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now reconsider the clues:

- The donkey lagged behind the rabbit.

So, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy that you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

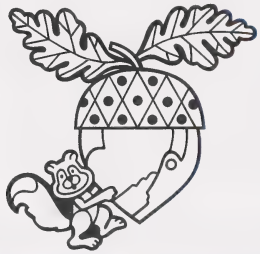
Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	F

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy was the robber.



Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example 1

Twelve less than a number is negative three. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is negative three.

Step 3: Solve the equation.

$$\begin{array}{rcl} n - 12 & = & -3 \\ +12 & & +12 \\ \hline n & = & 9 \end{array}$$

The number is 9.

Example 2

The second of two numbers is seven times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number.
Let $7n$ be the second number.

The second of two numbers is seven times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} & 7n \\ &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example 3

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n+20$	$5(n+20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 5n+10n+100 &= 850 \\ 15n+100 &= 850 \\ -100 \quad &-100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} &n+20 \\ &=50+20 \\ &=70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

Abundant number: number classification when the sum of the proper factors is greater than the number

Base: (of a power) the part of a power that shows what factor is multiplied by itself

Common factor: (of two or more numbers) a number that is a factor of each of the numbers

Common multiple: (of two or more numbers) a number that is a multiple of each of the numbers

Composite number: a number that has more than two different factors

Consecutive numbers: numbers written in order

Constant feature: (of a calculator) a feature of a calculator that allows a person to add, subtract, multiply, or divide repeatedly

Deficient number: number classification when the sum of the proper factors is less than the number

Dividend: in division, the number you are dividing

Dividing: the process of finding how many elements there are in a given number of groups where each group has the same number of elements; the process of finding how many groups there are of a specified number in a given number

Divisibility test: a test to quickly decide if a number is a factor of another number

Divisible: able to be divided so there is no remainder

Divisor: in division, the number you are dividing by; also called a **factor**

Exponent: (of a power) the part of a power that shows how many times the base is used as a factor

Factor: (noun) any one of the numbers used in multiplication to form a product; a whole number that exactly divides another whole number

Factor: (verb) to express a number as a product of its factors

Factored form: (of a number) a form of a number written as the product of factors

Factor tree: a diagram used to find the prime factorization of a number

Flow chart: a method for listing instructions

Greatest common factor: (of two or more numbers) the greatest of the common factors of the numbers

Inductive reasoning: the process of arriving at a conclusion by generalizing

Least common multiple: (of two or more numbers) the least of the common multiples of the numbers; the product of the common factors and the remaining factors

Multiple: (of a number) the product of the number and another number; the sum of adding the same number repeatedly

Multiple board: a learning aid that has the first ten multiples of a number on it

Multiplier: in multiplication, the number you are multiplying by

Multiplying: the process of finding the total number of elements in a given number of groups where each group has the same number of elements

"No" example: an example that shows a case that does not fit the definition

Perfect number: the number classification when the sum of the proper factors is equal to the number

Power: a compact way of showing the product of identical factors; consists of a base and an exponent

Power form: a form of a number written as a power; sometimes called **exponential form**

Prime factor: a factor that is a prime number

Prime factorization: the process of finding all the prime factors of a given number; an expression of a number as the product of its prime factors

Prime number: a number that has exactly two factors (the number 1 and itself)

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Product: the result of multiplication

Proper divisor: (of a number) a factor less than the number itself; also called a **proper factor**

Proper factor: (of a number) a factor less than the number itself; also called a **proper divisor**

Relatively prime: two numbers for which the greatest common factor is 1

Remainder: in division, the amount left over

Sieve of Eratosthenes: a method for sorting prime numbers

Standard form: the usual form of a number

T-table: a table made in the shape of the letter "T"

Twin prime: two prime numbers that have a difference of 2

Venn diagram: a special kind of diagram in which circles are used to represent sets of numbers

"Yes" example: an example that shows a case that fits the definition

Suggested Answers

Section 1: Activity 1

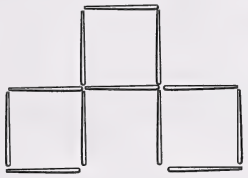
1. a. 4, 8, 12, 16, 20 **b.** 5, 10, 15, 20, 25

2. a. 8, 16, 24, 32, 40 **b.** 12, 24, 36, 48, 60
c. 23, 46, 69, 92, 115 **d.** 32, 64, 96, 128, 160

3. a. no **b.** no **c.** no **d.** yes **e.** yes **f.** no

Now Try This

4. You may use diagrams or actual toothpicks to help you solve this problem.



Section 1: Activity 2

1. a. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60

b. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90

2. a. 6, 12, (18), 24, 30, (36), 42, 48, (54), 60

9, (18), 27, (36), 45, (54), 63, 72, 81, 90

b. 6, 12, [18], 24, 30, (36), 42, 48, (54), 60

9, [18], 27, (36), 45, (54), 63, 72, 81, 90

3. a. 4, 8, [12], 16, 20, (24), 28, 32, (36), ...

6, [12], 18, (24), 30, (36), 42, ...

The first three common multiples of 4 and 6 are 12, 24, and 36.

The least common multiple is 12.

b. 8, 16, [24], 32, 40, (48), 56, 64, (72), 80, ...

12, [24], 36, (48), 60, (72), 84, 96, 108, 120, ...

The first three common multiples of 8 and 12 are 24, 48, and 72.

The least common multiple is 24.

c. 18, 36, 54, [72], 90, 108, 126, (144), 162, 180, 198, (216), ...

24, 48, [72], 96, 120, (144), 168, 192, (216), 240, 264, 288, ...

The first three common multiples of 18 and 24 are 72, 144, and 216.

The least common multiple is 72.

4. a. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60
6, 12, 18, 24, 30, 36, 42, 48, 54, 60
10, 20, 30, 40, 50, 60

The least common multiple of 4, 6, and 10 is 60.

b. 16, 32, 48, 64, 80, 96, 112, 128, 144
18, 36, 54, 72, 90, 108, 126, 144
24, 48, 72, 96, 120, 144

The least common multiple of 16, 18, and 24 is 144.

5. a. **Camilla:** 9, 18, 27, 36
Karen: 12, 24, 36

The least amount of time in which Camilla and Karen could cross the finish line together is 36 min.

b. **Camilla:** $36 \div 9 = 4$
Karen: $36 \div 12 = 3$

Camilla will have completed 4 laps and Karen will have completed 3 laps. So, Camilla will have completed more laps in 36 min.

$4 - 3 = 1$

Camilla will have completed 1 more lap than Karen.

6. a. **Plant 1:** 8, 16, 24, 32, 40
Plant 2: 12, 24, 36, 48, 60

On the 24th day both plants will again be watered on the same day. That date is January 24.

b. $365 \div 24 = 15$

Both plants will be watered on the same day 15 times in a year.

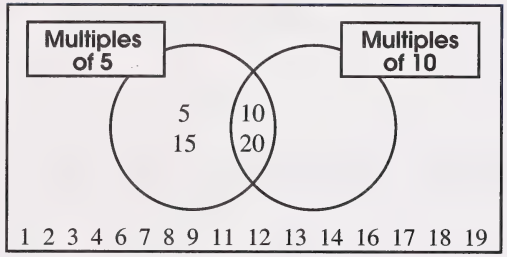
Now Try This

7. You can make an organized list of the possible combinations. The list may be organized in a table.

Quarters	Dimes	Nickels
0	0	16
0	1	14
0	2	12
0	3	10
0	4	8
0	5	6
0	6	4
0	7	2
0	8	0
1	0	11
1	1	9
1	2	7
1	3	5
1	4	3
1	5	1
2	0	6
2	1	4
2	2	2
2	3	0
3	0	1

Section 1: Activity 3

1.



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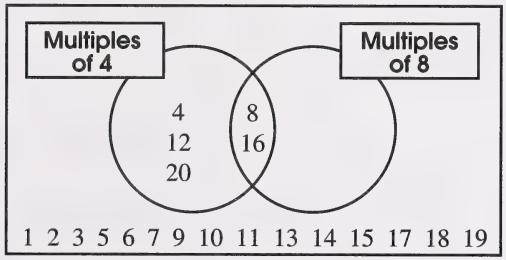

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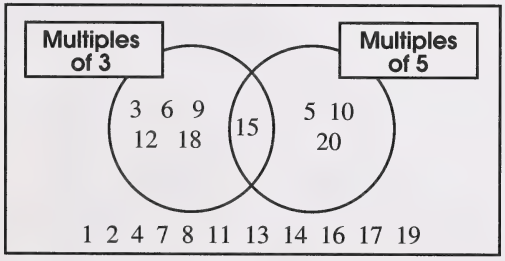

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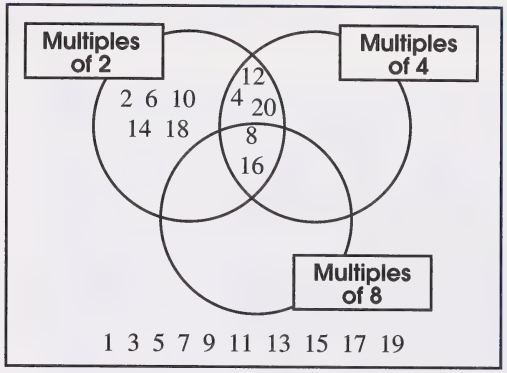
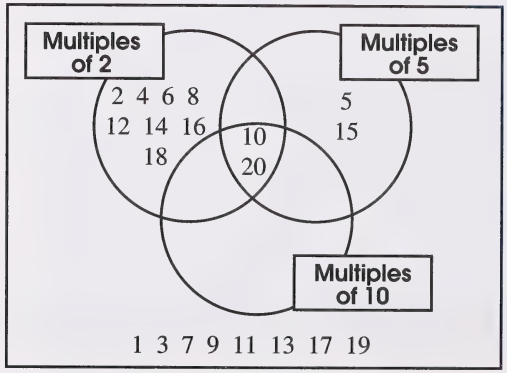


5.



Now Try This

6. You can solve this problem using the guess, check, and revise strategy.

To make it easier to guess and check, you can write the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 on small squares of paper.

There are several possible answers; two of the possible answers are given.

$$\begin{array}{r} 7\ 8\ 9 \\ +\ 2\ 4\ 6 \\ \hline 1\ 0\ 3\ 5 \end{array} \qquad \begin{array}{r} 2\ 4\ 6 \\ +\ 7\ 8\ 9 \\ \hline 1\ 0\ 3\ 5 \end{array}$$

Section 1: Follow-up Activities

Extra Help

1. **a.** 6 **b.** 30 **c.** 24 **d.** 12 **e.** 20 **f.** 24

2. **a.** 24 **b.** 12 **c.** 40 **d.** 18 **e.** 12 **f.** 30

3. The least common multiple of 10 and 12 is 60. Since $60 \div 12 = 5$ and $60 \div 10 = 6$, you should buy 5 packages of buns and 6 packages of wieners.

Enrichment

The Enrichment has no suggested answers for questions 1 to 3.

4. The number of members must be 1 greater than the least common multiple of 2, 3, 4, and 5. Since the LCM is 60, the least number of members that could have been in the marching band is 61.

5. The LCM of 36 and 45 is 180. Since 180 min = 3 h, the videos will start together at 8:45 A.M., 11:45 A.M., and 2:45 P.M.

Section 2: Activity 1

1. **a.**

$$\begin{array}{r} 435 \\ 3\overline{)1306} \\ \underline{12} \\ 10 \\ \underline{9} \\ 16 \\ \underline{15} \\ 1 \end{array}$$

Because there is a remainder, 3 is not a factor.

b.

$$\begin{array}{r} 3286 \\ 3\overline{)9858} \\ \underline{9} \\ 8 \\ \underline{6} \\ 25 \\ \underline{24} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

Because there is no remainder, 3 is a factor.

c.

$$\begin{array}{r} 1947 \\ 3\overline{)5842} \\ \underline{3} \\ 28 \\ \underline{27} \\ 14 \\ \underline{12} \\ 22 \\ \underline{21} \\ 1 \end{array}$$

Because there is a remainder, 3 is not a factor.

2. **a.**

$$\begin{array}{r} 2\ 9\ 0\ \text{R}3 \\ 4\overline{)11_3 6\ 3} \end{array}$$

Because there is a remainder, 4 is not a factor.

b.
$$4\overline{)7_{3}2\ 3_{3}2}\quad 1\ 8\ 0\ 8\ \text{R}0$$

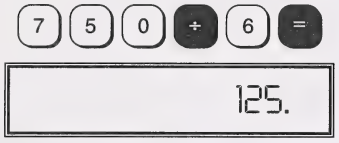
Because there is no remainder, 4 is a factor.

c.
$$4\overline{)5_{1}0_{2}1_{1}8}\quad 1\ 2\ 5\ 4\ \text{R}2$$

Because there is a remainder, 4 is not a factor.

3. **a.** no **b.** no **c.** yes

4.



The answer is a whole number. So, Jamaal can cut the roll into 6-cm pieces without any waste.

Now Try This

5. You can solve the problem by finding and applying a pattern.

Number of Students	Number of Games	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4
6	15	+5
7	21	+6
8	28	+7

Eight students would play 28 games of chess.

Section 2: Activity 2

1. All numbers divisible by 2 are even and all numbers not divisible by 2 are odd. So, the last digit of a number will help you decide if the number is divisible by 2. Numbers ending in 0, 2, 4, 6, or 8 are divisible by 2.

2. **a.** yes **b.** yes **c.** no

3. **a.** All numbers divisible by 5 end in 5 or 0. Numbers not divisible by 5 can end in 1, 2, 3, 4, 6, 7, 8, or 9. So, the last digit of a number will help you decide if the number is divisible by 5. Numbers ending in 5 or 0 are divisible by 5.

b. All numbers divisible by 10 end in 0. Numbers not divisible by 10 can end in 1, 2, 3, 4, 5, 6, 7, 8, or 9. So, the last digit of a number will help you decide if the number is divisible by 10. Numbers ending in 0 are divisible by 10.

4. **a.** yes **b.** yes **c.** no

5. **a.** no **b.** no **c.** yes

6. **a.** No, you cannot use the last digit of a number to decide if 4 or 8 is a factor. The "yes" examples of numbers divisible by 4 and 8 end in 0, 2, 4, 6, or 8. Some "no" examples also end in 0, 2, 4, 6, or 8.

b. Yes, you can use the last digit of a number to eliminate 4 or 8 as a factor. If a number is odd, the number is not divisible by 4 or 8.

7. a. The last two digits of numbers divisible by 4 are multiples of 4. The last two digits of numbers not divisible by 4 are not multiples of 4. You can conclude that if the last two digits of a number are divisible by 4, the number is divisible by 4.

b. The last three digits of numbers divisible by 8 are multiples of 8. The last three digits of numbers not divisible by 8 are not multiples of 8. You can conclude that if the last three digits of a number are divisible by 8, the number is divisible by 8.

8. a. yes **b.** no **c.** yes **d.** yes **e.** no **f.** no

9. a. yes **b.** no **c.** yes **d.** no **e.** yes **f.** yes

10. a. No, you cannot use the last digit of a number to decide if a number is divisible by 3. The "yes" examples of numbers divisible by 3 end in 1, 2, 3, 4, 5, 6, 7, 8, 9, or 0. The "no" examples of numbers also end in 1, 2, 3, 4, 5, 6, 7, 8, 9, or 0.

b. No, you cannot use the last digit of a number to decide if a number is divisible by 6. The "yes" examples of numbers divisible by 6 end in 0, 2, 4, 6, or 8. Some "no" examples also end in 0, 2, 4, 6, or 8.

c. No, you cannot use the last digit of a number to decide if a number is divisible by 9. The "yes" examples of numbers divisible by 9 can end with any digit. The "no" examples of numbers can also end with any digit.

d. You cannot use the last digit to eliminate 3 or 9 as a factor. However, you can use the last digit to eliminate 6 as a factor. If a number is odd, 6 cannot be a factor because numbers divisible by 6 are always even.

11. a. No, there is no relationship between divisibility by 3 and the last two or three digits. For example, 20 301 and 21 301 end in the same last three digits, but 20 301 is divisible by 3 and 21 301 is not divisible by 3.

b. No, there is no relationship between divisibility by 6 and the last two or three digits. For example, 42 132 and 25 132 end in the same last three digits, but 42 132 is divisible by 6 and 25 132 is not divisible by 6.

c. No, there is no relationship between divisibility by 9 and the last two or three digits. For example, 78 903 and 70 903 end in the same last three digits, but 78 903 is divisible by 9 and 70 903 is not divisible by 9.

12. a. Yes, the sum of the digits of each of the numbers in the first column is divisible by 3. No, none of the sums of the digits of any of the numbers in the last column is divisible by 3. You can conclude that if the sum of the digits of a number is divisible by 3, the number is divisible by 3.

b. In the column titled "Numbers Divisible by 6" each number is even and the sum of the digits of each number is divisible by 3. You can conclude that numbers divisible by 6 are even and have the sum of their digits divisible by 3.

c. In the column titled "Numbers Divisible by 9" the sum of the digits of each number is divisible by 9. You can conclude that if the sum of the digits of a number is divisible by 9, the number is divisible by 9.

13. **a.** no **b.** yes **c.** yes

14. **a.** no **b.** yes **c.** no

15. **a.** yes **b.** yes **c.** no

Now Try This

16. You can use a table to organize your response.

Numbers	Number of 3s	Number of 8s
100 to 110	1	1
111 to 120	1	1
121 to 130	2	1
131 to 140	10	1
141 to 150	1	1
151 to 160	1	1
161 to 170	1	1
171 to 180	1	2
181 to 190	1	10
191 to 199	1	1
Total	20	20

The computer will beep a total of 40 times.

Section 2: Activity 3

1. **a.**

30	
1	30
2	15
3	10
~~4~~	
5	6
~~6~~	~~5~~

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

b.

32	
1	32
2	16
~~3~~	
4	8
~~5~~	
~~6~~	
~~7~~	
~~8~~	~~4~~

The factors of 32 are 1, 2, 4, 8, 16, and 32.

c.

45	
1	45
~~2~~	
3	15
~~4~~	
5	9
~~6~~	
~~7~~	
~~8~~	
~~9~~	~~5~~

The factors of 45 are 1, 3, 5, 9, 15, and 45.

d.

70	
1	70
2	35
~~3~~	
~~4~~	
5	14
~~6~~	
7	10
~~8~~	
~~9~~	
~~10~~	~~7~~

The factors of 70 are 1, 2, 5, 7, 10, 14, 35, and 70.

2.

12	
1	12
2	6
3	4
~~4~~	~~3~~

Mike can arrange the butterflies in 1 row of 12, 2 rows of 6, 3 rows of 4, 4 rows of 3, 6 rows of 2, or 12 rows of 1.

3.

Number	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6
13	1, 13	2
14	1, 2, 7, 14	4
15	1, 3, 5, 15	4
16	1, 2, 4, 8, 16	5
17	1, 17	2
18	1, 2, 3, 6, 9, 18	6
19	1, 19	2
20	1, 2, 4, 5, 10, 20	6

4. **a.** The number 1 has exactly one factor.

b. The numbers 2, 3, 5, 7, 11, 13, 17, and 19 have exactly two factors.

c. The numbers 4 and 9 have exactly three factors.

d. The numbers 6, 8, 10, 14, and 15 have exactly four factors.

e. The number 16 has exactly five factors.

f. The numbers 12, 18, and 20 have exactly six factors.

Did You Know?

5.

Number	Sum of Proper Factors	Kind of Number
1	N/A	N/A
2	1	deficient
3	1	deficient
4	1+2=3	deficient
5	1	deficient
6	1+2+3=6	perfect
7	1	deficient
8	1+2+4=7	deficient
9	1+3=4	deficient
10	1+2+5=8	deficient
11	1	deficient
12	1+2+3+4+6=16	abundant
13	1	deficient
14	1+2+7=10	deficient
15	1+3+5=9	deficient
16	1+2+4+8=15	deficient
17	1	deficient
18	1+2+3+6+9=21	abundant
19	1	deficient
20	1+2+4+5+10=22	abundant

Section 2: Activity 4

1. a.

24	
①	24
②	⑫
③	8
④	⑥
~~5~~	
~~6~~	~~4~~

36	
①	36
②	18
③	⑫
④	9
~~5~~	
⑥	6
~~7~~	
~~8~~	
~~9~~	~~4~~

To list the common factors of 24 and 36, note the circled factors as you read down the left-hand side of the T-table and up the right-hand side.

The common factors of 24 and 36 are 1, 2, 3, 4, 6, and 12.

b.

12	
①	12
②	⑥
③	4
~~4~~	~~3~~

18	
①	18
②	9
③	⑥
~~4~~	
~~5~~	
~~6~~	~~3~~

The common factors of 12 and 18 are 1, 2, 3, and 6.

c.

30	
1	30
2	15
3	10
4	
5	6
6	5

75	
1	75
2	
3	25
4	
5	15
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	5

The common factors of 30 and 75 are 1, 3, 5, and 15.

2. a.

6	
1	6
2	3
3	2

12	
1	12
2	6
3	4
4	3

The common factors of 6 and 12 are 1, 2, 3, and 6.

The greatest common factor is 6.

b.

30	
1	30
2	15
3	10
4	
5	6
6	5

45	
1	45
2	
3	15
4	
5	9
6	
7	
8	
9	5

The common factors of 30 and 45 are 1, 3, 5, and 15.

The greatest common factor is 15.

c.

42	
1	42
2	21
3	14
4	
5	
6	7
7	6

56	
1	56
2	28
3	
4	14
5	
6	
7	8
8	7

The common factors of 42 and 56 are 1, 2, 7, and 14.

The greatest common factor is 14.

3. a.

54	
①	54
2	27
③	18
~~4~~	
~~5~~	
6	⑨
~~7~~	
~~8~~	
~~9~~	~~6~~

81	
①	81
~~2~~	
③	27
~~4~~	
~~5~~	
~~6~~	
~~7~~	
~~8~~	
⑨	9

63	
①	63
~~2~~	
③	21
~~4~~	
~~5~~	
~~6~~	
7	⑨
~~8~~	
~~9~~	~~7~~

The common factors of 54, 81, and 63 are 1, 3, and 9.

The greatest common factor is 9.

b.

8	
①	8
②	④
~~3~~	
~~4~~	~~2~~

16	
①	16
②	8
~~3~~	
④	4
~~5~~	
~~6~~	
~~7~~	
~~8~~	~~2~~

20	
①	20
②	10
~~3~~	
④	5
~~5~~	~~4~~

The common factors of 8, 16, and 20 are 1, 2, and 4.

The greatest common factor is 4.

c.

24	
①	24
②	12
③	8
4	⑥
~~5~~	
~~6~~	~~4~~

42	
①	42
②	21
③	14
~~4~~	
~~5~~	
⑥	7
~~7~~	~~6~~

48	
①	48
②	24
③	16
4	12
~~5~~	
⑥	8
~~7~~	
~~8~~	~~6~~

The common factors of 24, 42, and 48 are 1, 2, 3, and 6.

The greatest common factor is 6.

4.

28	
①	28
②	14
~~3~~	
④	7
~~5~~	
~~6~~	
~~7~~	~~4~~

72	
①	72
②	36
3	24
④	18
~~5~~	
6	12
~~7~~	
8	9
~~9~~	~~8~~

96	
①	96
②	48
3	32
④	24
~~5~~	
6	16
~~7~~	
8	12
~~9~~	
~~10~~	
~~11~~	
~~12~~	~~8~~

The common factors are 1, 2, and 4.

The greatest common factor is 4.

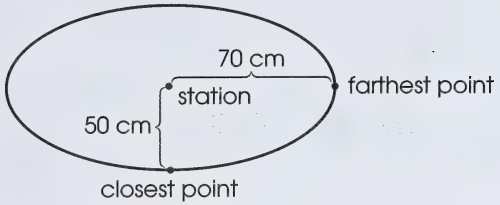
So, the most that each jawbreaker could cost is 4¢.

Now Try This

5. You can make a related but simpler problem to help you understand the problem.

Example

A toy train circles a railway station in an elliptical path. At its farthest point the train is 70 cm from the station. At its closest point the train is 50 cm from the station. How much farther from the station is the train when it is at its farthest point, compared to when it is at its closest point?



$70 - 50 = 20$

The train is 20 cm farther from the station.

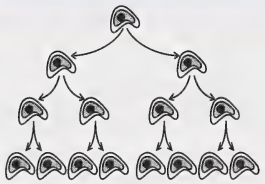
From this simpler model you can see how to solve the original problem.

$$109\,000\,000 - 107\,000\,000 = 2\,000\,000$$

Venus is 2 000 000 km farther from the Sun at the farthest point in its orbit as compared to the closest point in its orbit.

Section 2: Activity 5

1.

Cells	Number of Splits	Number of Animals	Pattern
	0	1	
	1	2	×2
	2	4	×2
	3	8	×2

There are twice as many animals after each successive split:

- After 1 split there are 2 animals.
- After 2 splits there are 4 animals; $2 \times 2 = 4$.
- After 3 splits there are 8 animals; $2 \times 2 \times 2 = 8$.

2. a. After 4 splits there will be 16 animals; $2 \times 2 \times 2 \times 2 = 16$.

b. After 5 splits there will be 32 animals; $2 \times 2 \times 2 \times 2 \times 2 = 32$.

c. After 6 splits there will be 64 animals; $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.

d. After 7 splits there will be 128 animals; $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$.

3. a. $3 \times 3 \times 3 \times 3 \times 3 = 3^5$ **b.** $5 \times 5 \times 5 = 5^3$

c. $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

4. a. $5^7 = 5\times5\times5\times5\times5\times5\times5$ **b.** $3^4 = 3\times3\times3\times3$

c. $7^2 = 7\times7$

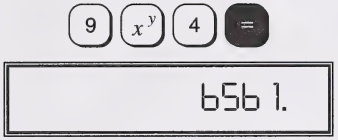
5. a. This was done as an example.

b. $3^4 = 3\times3\times3\times3$
$= 9\times3\times3$
$= 27\times3$
$= 81$

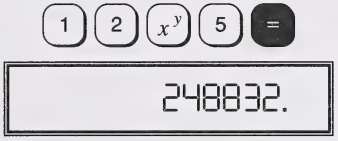
c. $5^3 = 5\times5\times5$
$= 25\times5$
$= 125$

d. $7^4 = 7\times7\times7\times7$
$= 49\times7\times7$
$= 343\times7$
$= 2401$

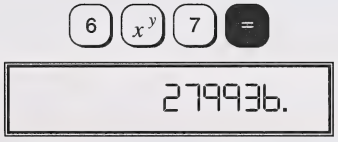
6. a.



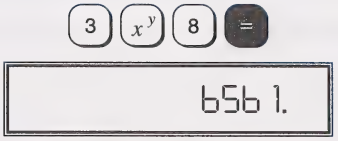
b.



c.



d.



Section 2: Follow-up Activities

The Extra Help and Enrichment have no suggested answers.

Section 3: Activity 1

1. a. The number 21 is composite; 21 has more than two factors. The factors are 1, 3, 7, and 21.

b. The number 17 is prime; 17 has exactly two factors. The factors are 1 and 17.

c. The number 39 is composite; 39 has more than two factors. The factors are 1, 3, 13, and 39.

d. The number 51 is composite; 51 has more than two factors. The factors are 1, 3, 17, and 51.

e. The number 73 is prime; 73 has exactly two factors. The factors are 1 and 73.

f. The number 45 is composite; 45 has more than two factors. The factors are 1, 3, 5, 9, 15, and 45.

g. The number 69 is composite; 69 has more than two factors. The factors are 1, 3, 23, and 69.

h. The number 83 is prime; 83 has exactly two factors. The factors are 1 and 83.

2. a. The first two consecutive prime numbers are 2 and 3.

b. The first string of three consecutive composite numbers is 8, 9, and 10.

c. The first string of five consecutive composite numbers is 24, 25, 26, 27, and 28.

3. The twin primes between 1 and 50 are 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, and 41 and 43.

4. The first four prime numbers are 2, 3, 5, and 7.

$2+3+5+7=17$

The sum of the first four prime numbers is 17. Yes, 17 is a prime number.

5. Four pairs of prime numbers where each pair has a sum of 50 are 3 and 47, 7 and 43, 13 and 37, and 19 and 31.

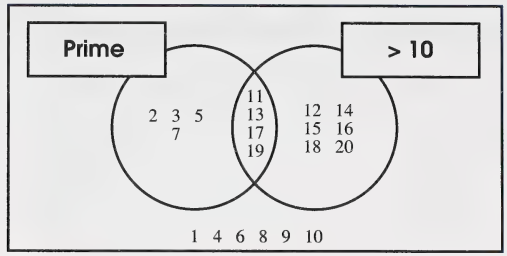
6. Answers may vary. Here are some possible answers.

a. $6=3+3$	**b.** $8=3+5$	**c.** $10=5+5$
d. $12=5+7$	**e.** $14=7+7$	**f.** $16=3+13$
g. $18=7+11$	**h.** $20=7+13$	

7.

a. $8=2+3+3$	**b.** $9=3+3+3$	**c.** $10=2+3+5$
d. $11=3+3+5$	**e.** $12=2+3+7$	**f.** $13=3+5+5$
g. $14=2+5+7$	**h.** $15=2+2+11$	

8.



Section 3: Activity 2

1. a.

15	
1	15
③	⑤
~~4~~	
~~5~~	~~3~~

The prime factors of 15 are 3 and 5.

b.

10	
1	10
②	⑤
~~3~~	
~~4~~	
~~5~~	~~2~~

The prime factors of 10 are 2 and 5.

c.

16	
1	16
②	8
~~3~~	
4	4
~~5~~	
~~6~~	
~~7~~	
~~8~~	~~2~~

The prime factor of 16 is 2.

d.

24	
1	24
②	12
③	8
4	6
~~5~~	
~~6~~	~~4~~

The prime factors of 24 are 2 and 3.

2. a.

38

2 × 19

$38 = 2 \times 19$

b.

42

2 × 21

2 × 3 × 7

$42 = 2 \times 3 \times 7$

c.

45

3 × 15

3 × 3 × 5

$45 = 3 \times 3 \times 5$

Note: In question 2, the factor trees may vary, but the resulting prime factorizations will be the same.

3. a.

2	100
2	50
5	25
5	5
	1

$100 = 2 \times 2 \times 5 \times 5$

b.

2	108
2	54
3	27
3	9
3	3
	1

$108 = 2 \times 2 \times 3 \times 3 \times 3$

c.

2	110
5	55
11	11
	1

$110 = 2 \times 5 \times 11$

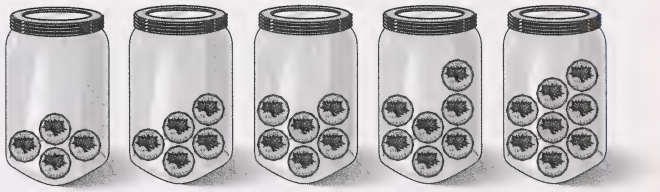
d.

3	99
3	33
11	11
	1

$99 = 3 \times 3 \times 11$

Now Try This

4. You may use a diagram or actual objects to help you solve this problem.



$4 + 5 + 6 + 7 + 8 = 30$

There are 30 pennies in all.

Section 3: Activity 3

1. Note: In this question, the factor trees may vary, but the resulting prime factorizations will be the same.

a.

24

4 × 6

2 × 2 × 2 × 3

36

6 × 6

2 × 3 × 2 × 3

$24 = 2 \times 2 \times 2 \times 3$

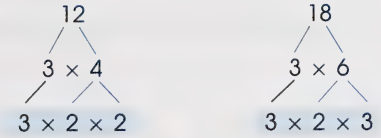
$36 = 2 \times 2 \times 3 \times 3$

$GCF = 2 \times 2 \times 3$

$= 12$

The greatest common factor of 24 and 36 is 12.

b.



$12 = 2 \times 2 \times 3$

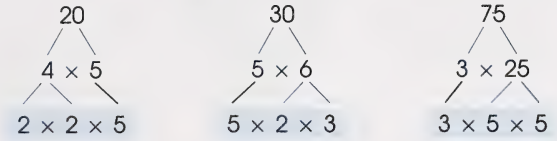
$18 = 2 \times 3 \times 3$

$GCF = 2 \times 3$

$= 6$

The greatest common factor of 12 and 18 is 6.

c.



$20 = 2 \times 2 \times 5$

$30 = 2 \times 3 \times 5$

$75 = 3 \times 5 \times 5$

The only complete column has 5s in it.

$GCF = 5$

The greatest common factor of 20, 30, and 75 is 5.

2. a.

2 | 48	2 | 64
2 | 24	2 | 32
2 | 12	2 | 16
2 | 6	2 | 8
3 | 3	2 | 4
1	2 | 2
	1

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$GCF = 2 \times 2 \times 2 \times 2$

$= 16$

The greatest common factor of 48 and 64 is 16.

b.

2 | 54	3 | 63
3 | 27	3 | 21
3 | 9	7 | 7
3 | 3	1
1	

$54 = 2 \times 3 \times 3 \times 3$

$63 = 3 \times 3 \times 7$

$GCF = 3 \times 3$

$= 9$

The greatest common factor of 54 and 63 is 9.

c.

12	18	30
2 | 12	2 | 18	2 | 30
2 | 6	3 | 9	3 | 15
3 | 3	3 | 3	5 | 5
1	1	1

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$

$GCF = 2 \times 3$
$= 6$

The greatest common factor of 12, 18, and 30 is 6.

Now Try This

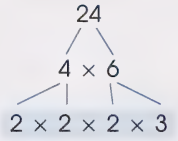
3. Make a diagram and work backwards from top to bottom.

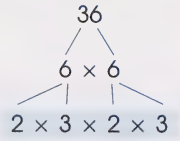
1	
2	
3	
4	
5	
6	
7	
8	
36	

Kris should begin with 8 cans on the bottom row.

Section 3: Activity 4

1. a.



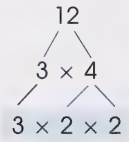


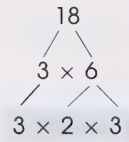
$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 3 \times 3$
$= 72$

The least common multiple of 24 and 36 is 72.

b.



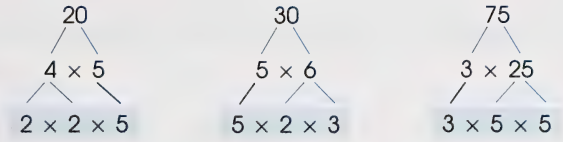


$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 3 \times 3$
$= 36$

The least common multiple of 12 and 18 is 36.

c.



$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$
$75 = 3 \times 5 \times 5$

$LCM = 2 \times 2 \times 3 \times 5 \times 5$
$= 300$

The least common multiple of 20, 30, and 75 is 300.

2. a.

2 | 54	3 | 81
3 | 27	3 | 27
3 | 9	3 | 9
3 | 3	3 | 3
1	1

$54 = 2 \times 3 \times 3 \times 3$
$81 = 3 \times 3 \times 3 \times 3$

$LCM = 2 \times 3 \times 3 \times 3 \times 3$
$= 162$

The least common multiple of 54 and 81 is 162.

b.

2 | 48	2 | 80
2 | 24	2 | 40
2 | 12	2 | 20
2 | 6	2 | 10
3 | 3	5 | 5
1	1

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$80 = 2 \times 2 \times 2 \times 2 \times 5$

$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
$= 240$

The least common multiple of 48 and 80 is 240.

c.

2 | 12	2 | 18	2 | 30
2 | 6	3 | 9	3 | 15
3 | 3	3 | 3	5 | 5
1	1	1

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$

$LCM = 2 \times 2 \times 3 \times 3 \times 5$
$= 180$

The least common multiple of 12, 18, and 30 is 180.

Now Try This

3. One way to solve this problem is to use the guess, check, and revise method. This method is illustrated using the following table.

Guess	Number of \$0.40 Fudge	Number of \$0.60 Fudge	Number of \$0.30 Fudge	Test
1	4	4	7	$4\times0.40=1.60$ $4\times0.60=2.40$ $7\times0.30=2.10$ Total 6.10
2	5	5	5	$5\times0.40=2.00$ $5\times0.60=3.00$ $5\times0.30=1.50$ Total 6.50
3	6	6	3	$6\times0.40=2.40$ $6\times0.60=3.60$ $3\times0.30=0.90$ Total 6.90

The third guess gives the proper total. So, Yves bought three pieces of the \$0.30 fudge.

Section 3: Follow-up Activities

Extra Help

1. Method 1: Using Repeated Division

$$\begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$18 = 2 \times 3 \times 3$

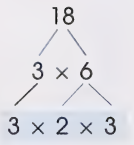
$36 = 2 \times 2 \times 3 \times 3$

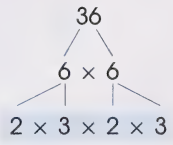
$GCF = 2\times3\times3 = 18$

$LCM = 2\times2\times3\times3 = 36$

The GCF of 18 and 36 is 18; the LCM is 36.

Method 2: Using Factor Trees





$18 = 2 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$GCF = 2\times3\times3 = 18$

$LCM = 2\times2\times3\times3 = 36$

The GCF of 18 and 36 is 18; the LCM is 36.

2. Method 1: Using Repeated Division

2	44	2	52
2	22	2	26
11	11	13	13
	1		1

$44 = 2 \times 2 \times 11$

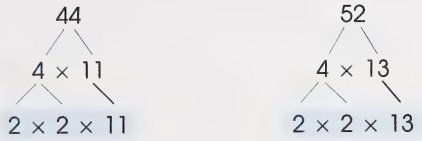
$52 = 2 \times 2 \times 13$

$GCF = 2 \times 2 = 4$

$LCM = 2 \times 2 \times 11 \times 13 = 572$

The GCF of 44 and 52 is 4; the LCM is 572.

Method 2: Using Factor Trees



$44 = 2 \times 2 \times 11$

$52 = 2 \times 2 \times 13$

$GCF = 2 \times 2 = 4$

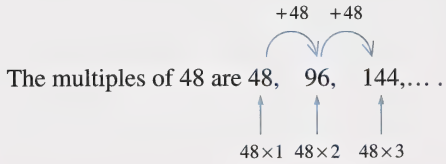
$LCM = 2 \times 2 \times 11 \times 13 = 572$

The GCF of 44 and 52 is 4; the LCM is 572.

Enrichment

1. **a.** No, the greatest common factor of 21 and 36 is 3.
b. Yes, the greatest common factor of 87 and 53 is 1.
c. Yes, the greatest common factor of 45 and 38 is 1.
d. No, the greatest common factor of 21 and 93 is 3.
e. Yes, the greatest common factor of 74, 26, and 21 is 1.
f. Yes, the greatest common factor of 8, 21, and 36 is 1.

2. Only multiples of 48 will have 48 as a factor.



The lowest three numbers with the greatest common factor of 48 are 48, 96, and 144.

Did You Know?

3. Start with 23 and subtract 8 to get 15. Then divide by 5 to get 3. Finally, multiply by 6 to get 18.

The number is 18.

4. Start with 40 and add 2 to get 42. Then divide by 7 to get 6. Next, multiply by 4 to get 24. Then, subtract 3 to get 21. Finally, subtract 10 to get 11.

The number is 11.

2	4	6	8	10	12	14	16	18	20

3	6	9	12	15	18	21	24	27	30

4	8	12	16	20	24	28	32	36	40

5	10	15	20	25	30	35	40	45	50

6	12	18	24	30	36	42	48	54	60

7	14	21	28	35	42	49	56	63	70

8	16	24	32	40	48	56	64	72	80

9	18	27	36	45	54	63	72	81	90

10	20	30	40	50	60	70	80	90	100

12	24	36	48	60	72	84	96	108	120

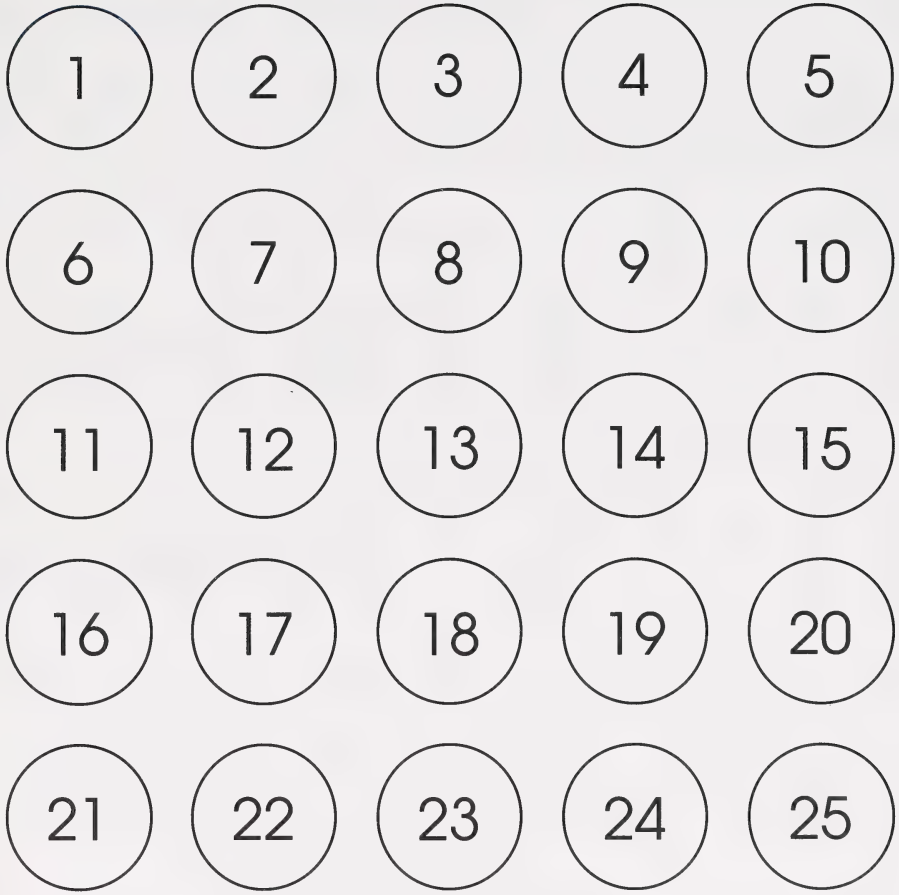
"Multiple Tick-Tack-Toe" Game Board

14	20	6	30
9	4	15	3
1	10	5	12
8	18	24	2

"The LCM Game" Board

12	20	6	30
9	4	15	3
1	10	5	12
6	30	20	2

Disks for "Factors" Game



"Place Roll" Game Board

5	3	7	PRIME
2	6	8	17
PRIME	21	4	16
13	9	PRIME	26

13	6	9	2
5	10	4	7
11	8	12	1
16	15	14	3

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